

Superconductivity - Overview

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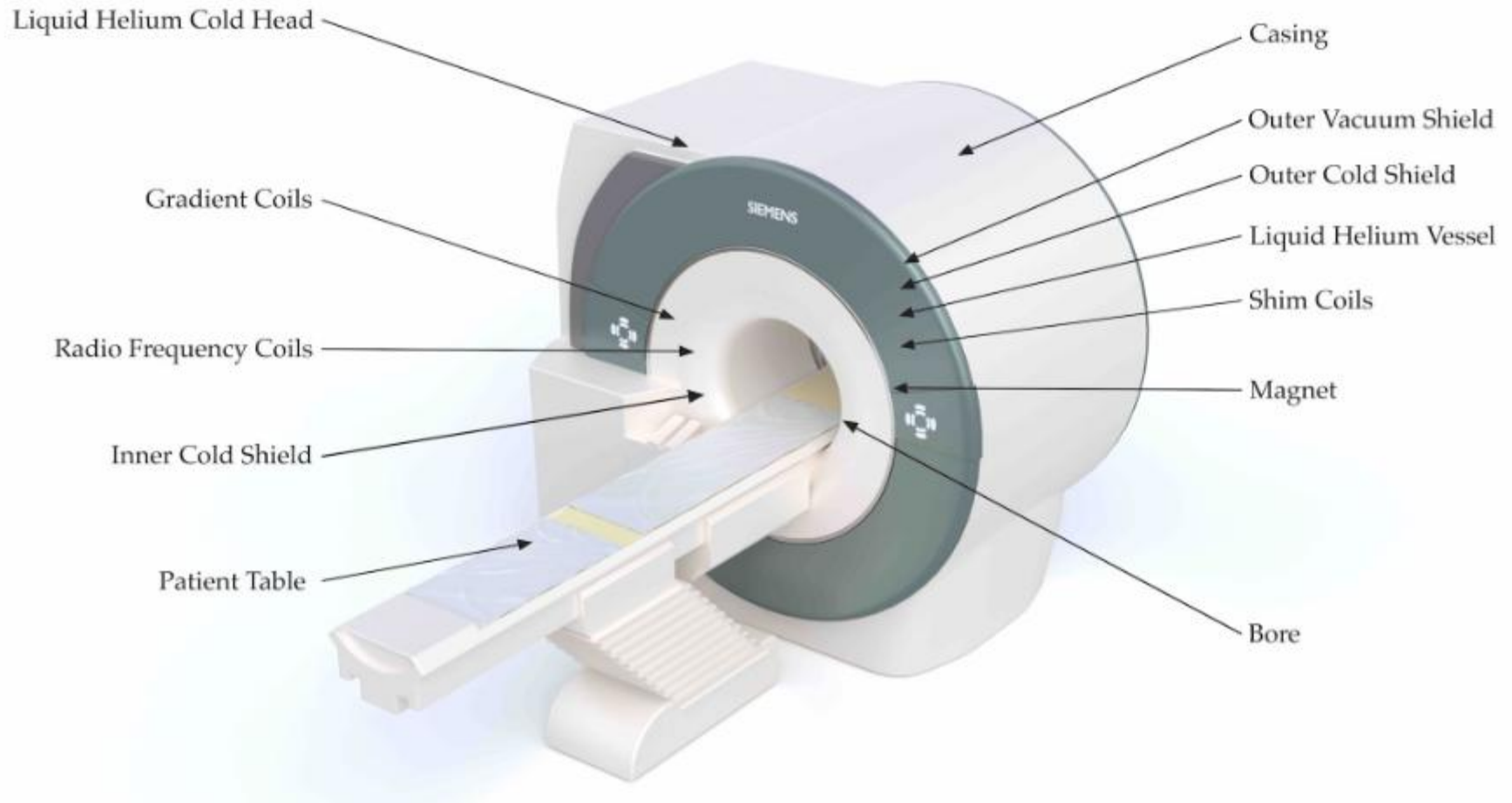


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Magnetic Resonance Imaging (MRI)



<https://socal-engineer.com/engineering-blog/2023/6/23/mri-technologies-overview>

[MRI Technologies Overview — So-Cal Engineer](#)



TM

Superconducting magnets for MRI

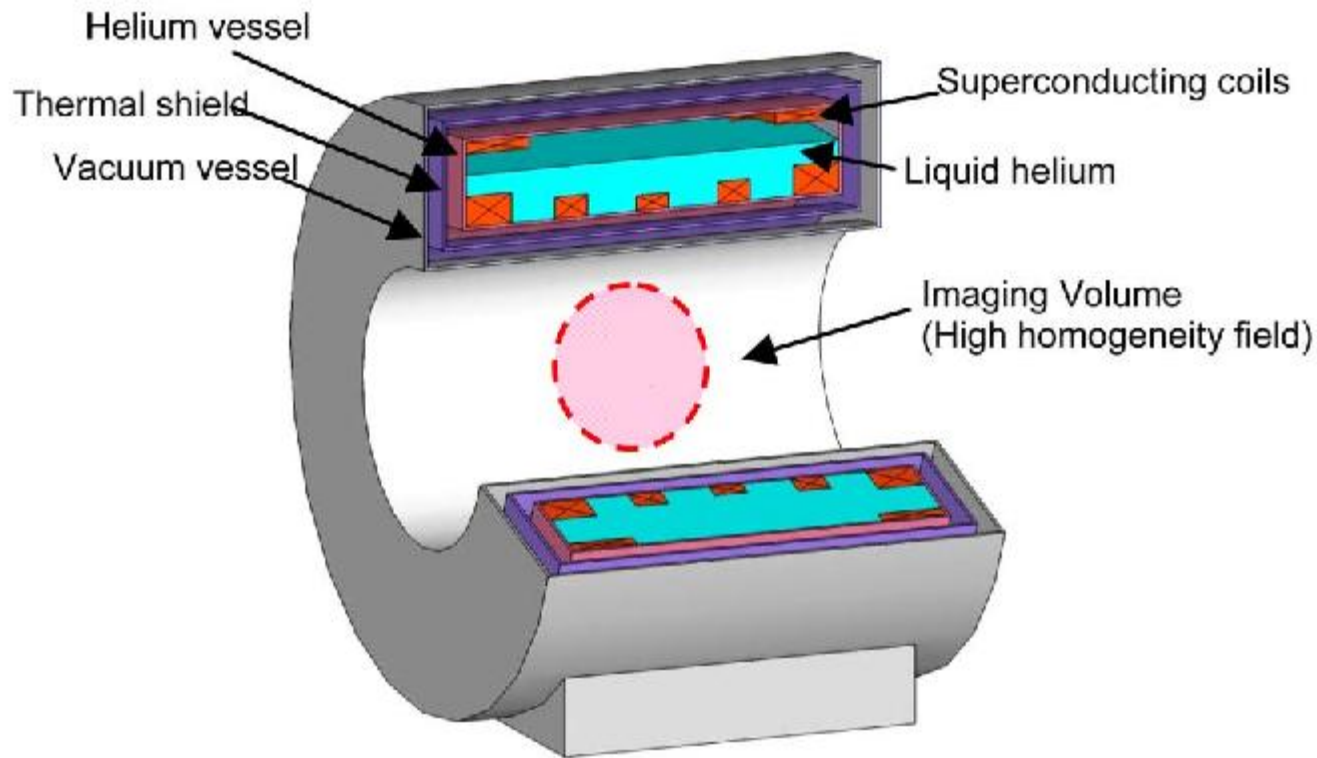


Fig. 2. Cross-sectional view of the magnet.

Published in IEEE transactions on applied superconductivity 2014

Super-Stable Superconducting MRI Magnet Operating for 25 Years

Shunji Yamamoto

Katsumi Konii

H. Tanabe

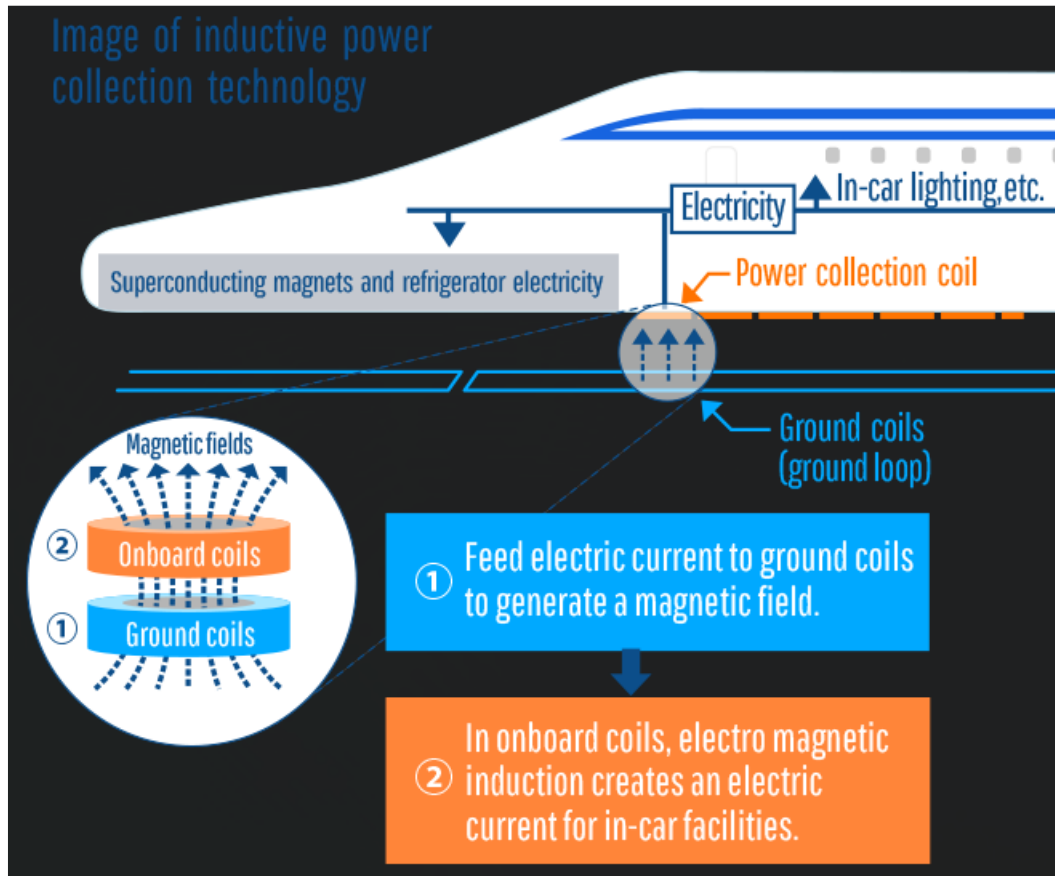
S. Yokoyama

T. Matsuda

T. Yamada



Superconducting-magnet levitation train



The [L0 Series](#), a prototype vehicle based on SCMaglev technology, holds the record for fastest crewed rail vehicle with a record speed of 603 km/h (375 mph).

Time Urbana-Chicago: only ~25 min

The SCMaglev system uses an [electrodynamic suspension](#) (EDS) system. The train's [bogies](#) have [superconducting](#) magnets installed, and the guideways contain two sets of metal coils.

Importance of superconductivity: Qubits and modern quantum computers are made of superconductors

IBM Q

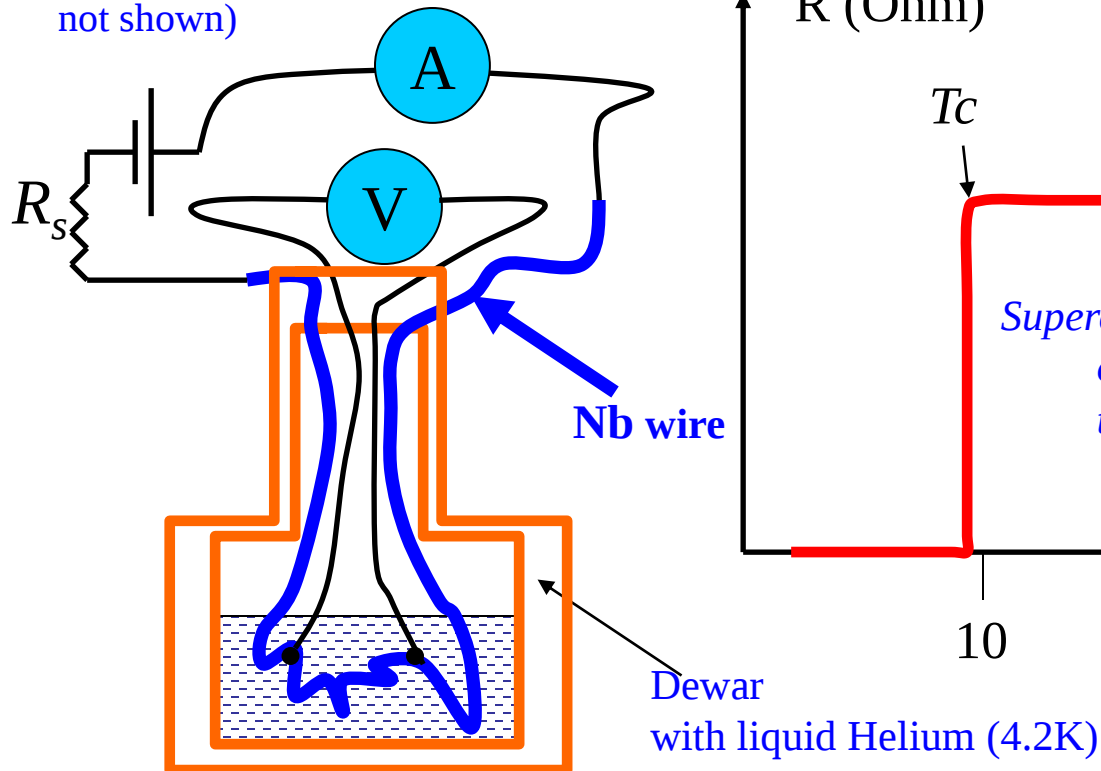


How to measure superconducting transitions

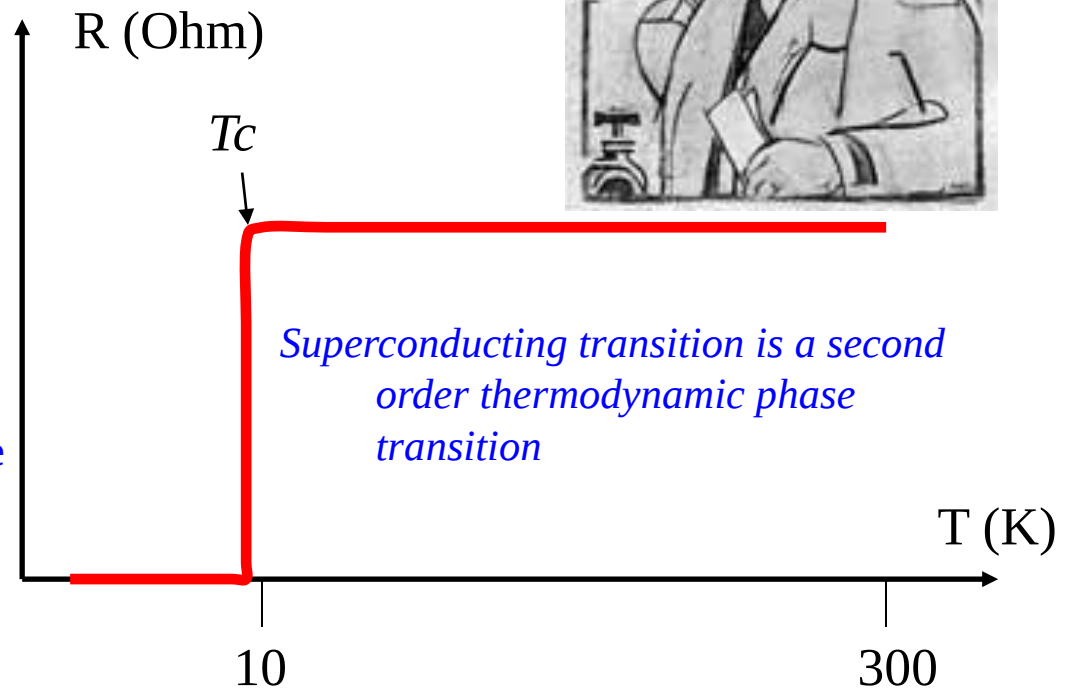
Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature" (H. K. O. 1911)

How to observe superconductivity

1. Take Nb (niobium) wire
2. Connect to a voltmeter and a current source
3. Immerse into helium Dewar ($T=4.2$ K boiling point)
4. Measure electrical resistance (R) versus the temperature (T) (thermometer is not shown)

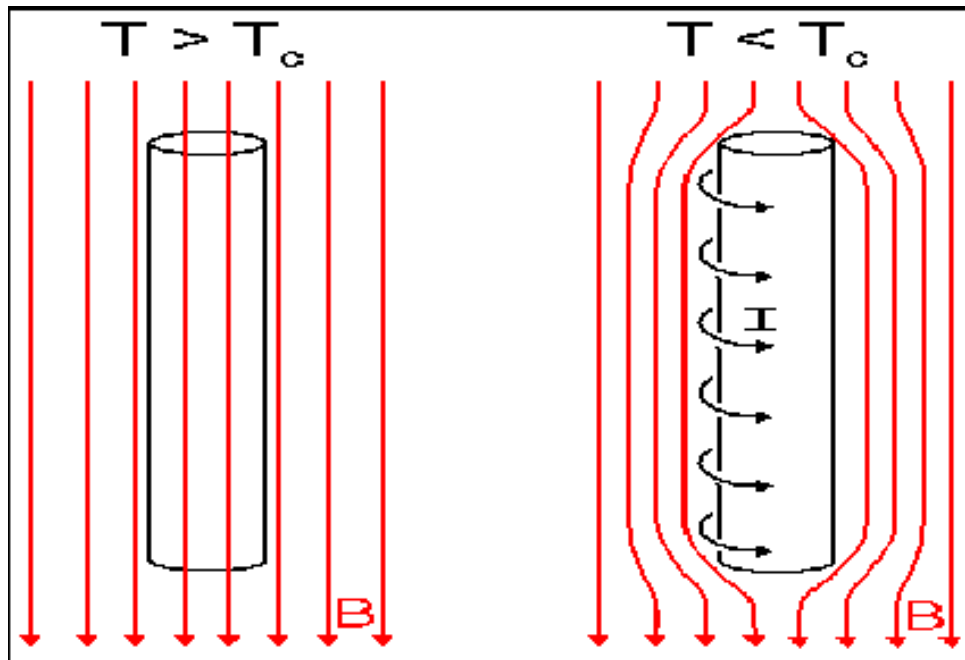


Heike Kamerling Onnes





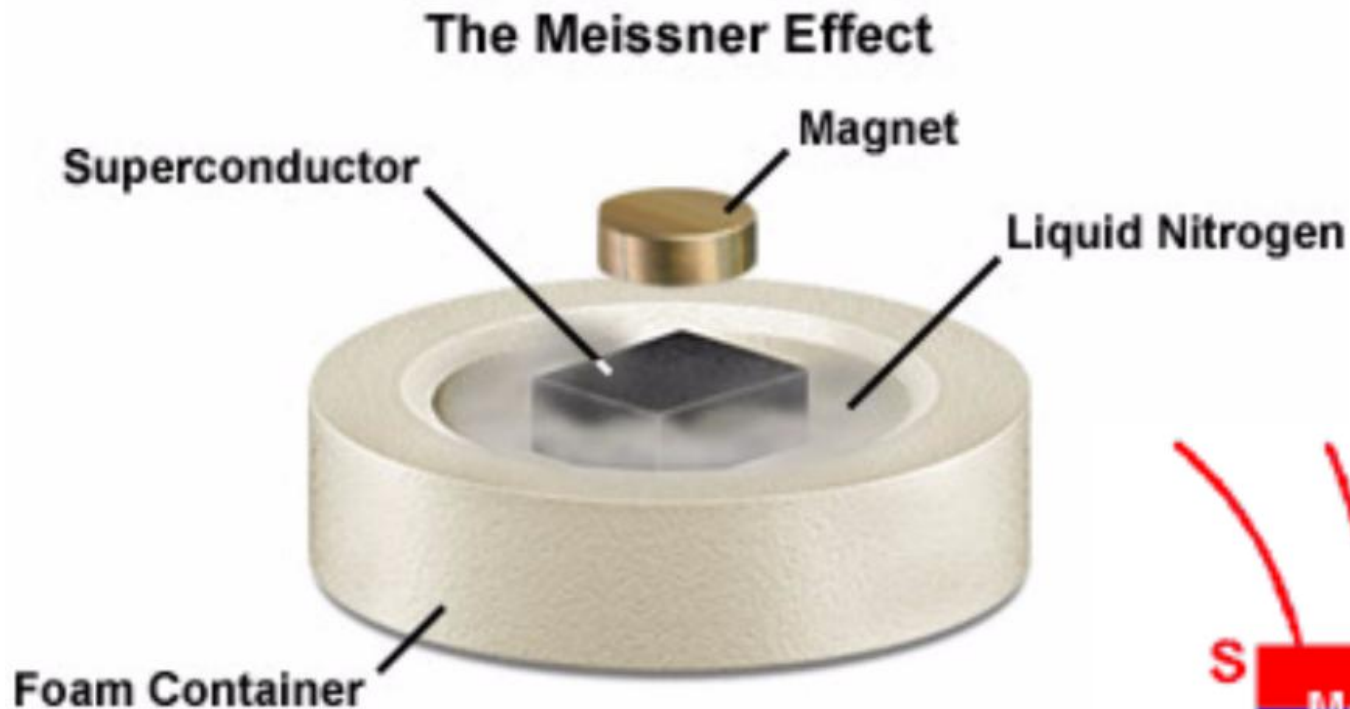
Meissner effect – the key signature of superconductivity



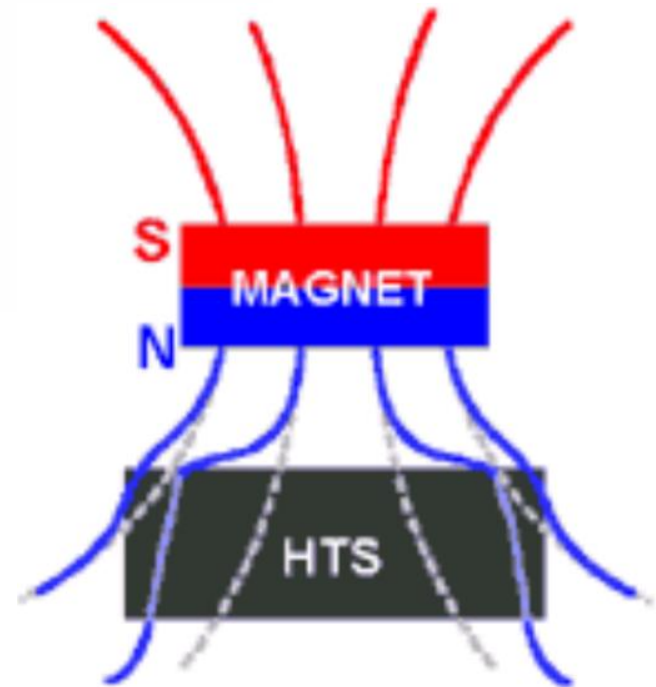
Theory of superconductivity:
“BCS” – due to Bardeen, Cooper and Schrieffer

Formula	T_c (K)	H_c (T)	Type	BCS
<i>Elements</i>				
Al	1.20	0.01	I	yes
Cd	0.52	0.0028	I	yes
Diamond:B	11.4	4	II	yes
Ga	1.083	0.0058	I	yes
Hf	0.165		I	yes
α -Hg	4.15	0.04	I	yes
β -Hg	3.95	0.04	I	yes
In	3.4	0.03	I	yes
Ir	0.14	0.0016 ^[7]	I	yes
α -La	4.9		I	yes
β -La	6.3		I	yes
Mo	0.92	0.0096	I	yes
Nb	9.26	0.82	II	yes
Os	0.65	0.007	I	yes

Interesting phenomenon: Magnetic levitation



Levitation is the process by which an object is held aloft, without mechanical support, in a stable position.



BCS Theory

- the origin of superconductivity

Bardeen Cooper and Schrieffer derived two expressions that describe the mechanism that causes superconductivity,

$$|\Delta| = 2\hbar\omega_D \exp\left[-\frac{1}{N(0)V}\right]$$
$$k_B T_c = 1.14\hbar\omega_D \exp\left[-\frac{1}{N(0)V}\right]$$

where T_c is the critical temperature, Δ is a constant energy gap around the Fermi surface, $N(0)$ is the density of states and V is the strength of the coupling.

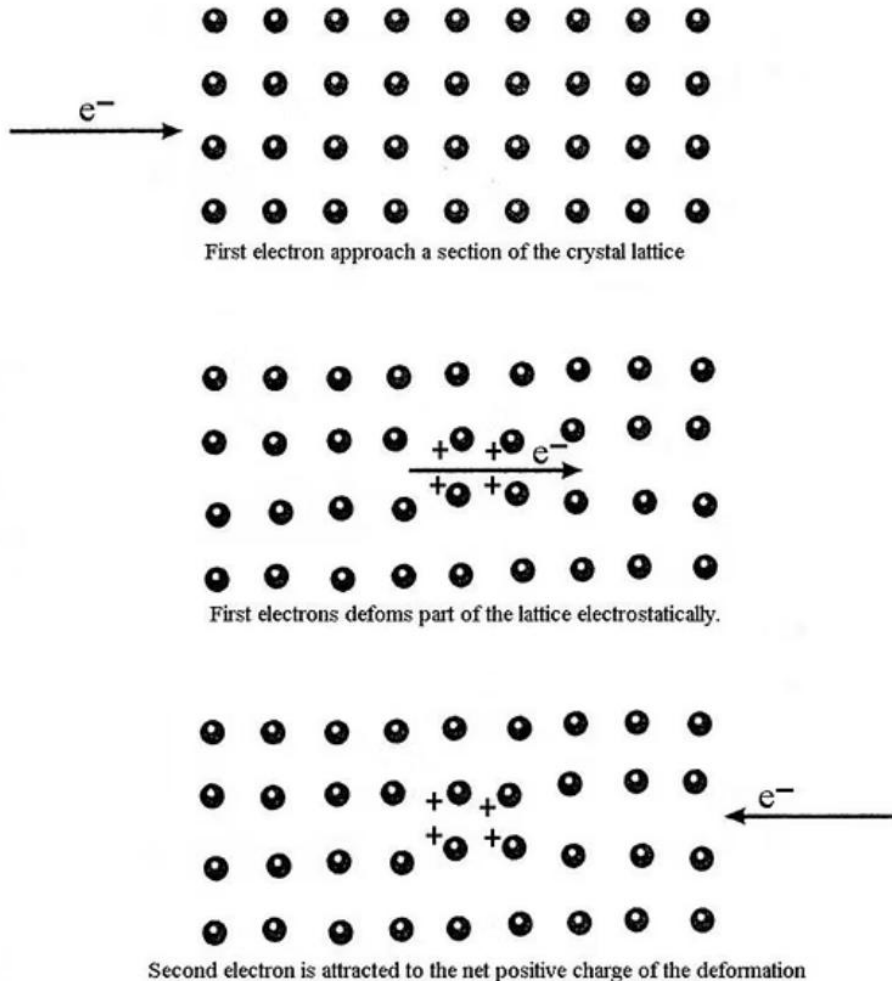


Figure 12.3: John Bardeen, Leon Cooper, and J. Robert Schrieffer.

Near T_c :
$$\Delta(T) = 1.734 \Delta(0) \left(1 - \frac{T}{T_c}\right)^{1/2} \simeq 3.06 k_B T_c \left(1 - \frac{T}{T_c}\right)^{1/2}$$

Attraction between the electrons is needed for superconductivity. It is provided by phonons.

Debye Frequency ω_D



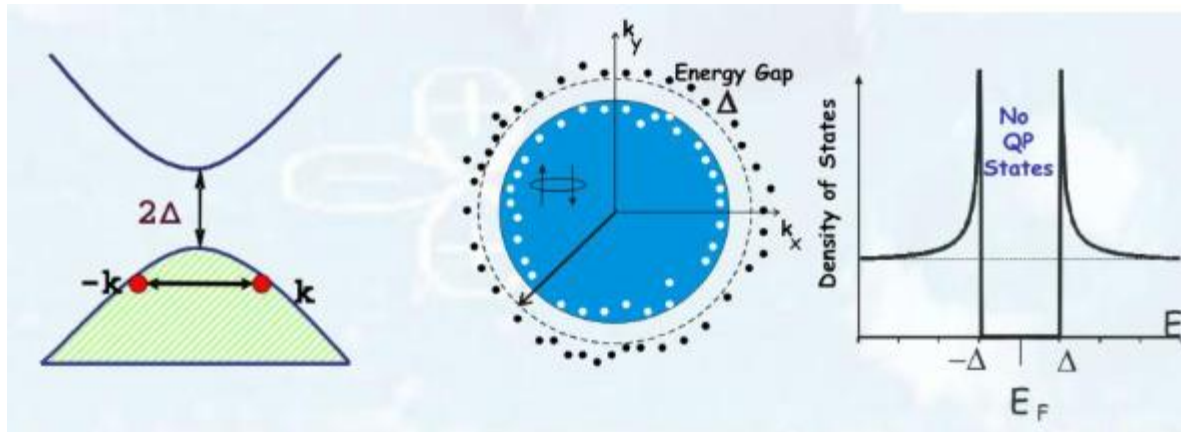
The Debye frequency represents the highest vibrational frequency of the atoms in the crystal lattice of the superconductor. There are no phonon modes above this frequency. Thus, Debye frequency is the upper limit of the phonon spectrum in the Debye model of specific heat.

<https://www.physicspower.com/post/bcs-theory>

BCS theory

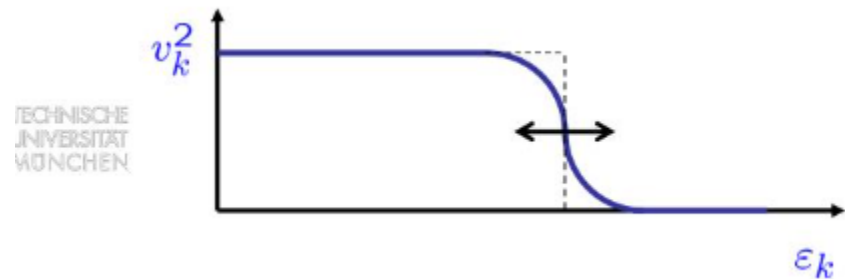
- Gap and order parameter

<https://phys.uri.edu/~nigh/Leuven-2011/henley/7.3.pdf>



$$|BCS\rangle = \prod_k (u_k + v_k a_k^\dagger a_{-k}^\dagger) |-\rangle$$

$$u_k^2 + v_k^2 = 1$$



BCS theory

- Gap and order parameter

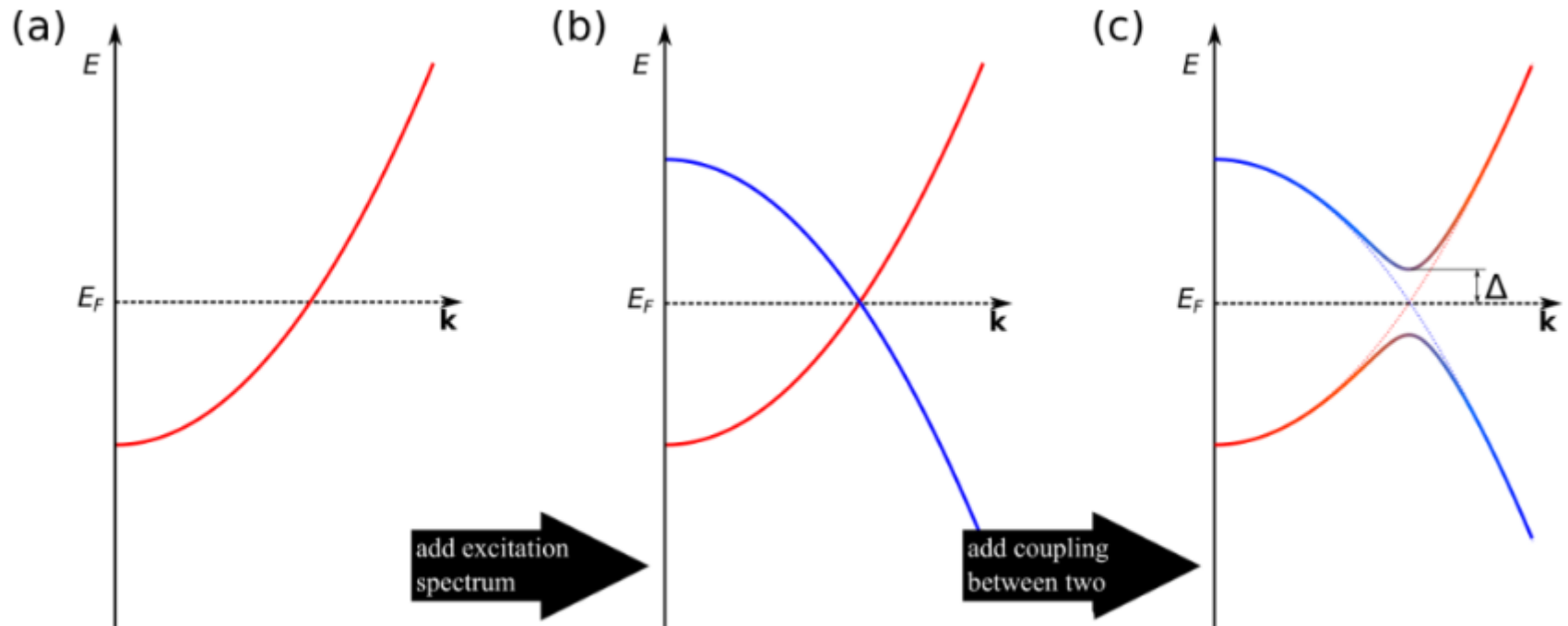
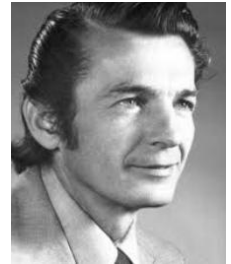


Figure 1: BCS theory: example of electron and hole bands coupling. (a) a parabolic electron-like band, (b) add the excitation hole-like band, (c) superconducting gap opens when the electron-like and hole-like bands are coupled by an interaction Δ .

Energy gap in superconductors: Giaever junction experiment

Near T_c :

$$\Delta(T) = 1.734 \Delta(0) \left(1 - \frac{T}{T_c}\right)^{1/2} \simeq 3.06 k_B T_c \left(1 - \frac{T}{T_c}\right)^{1/2}$$



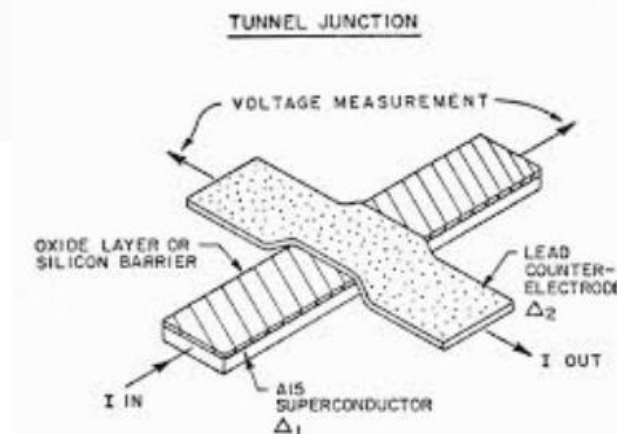
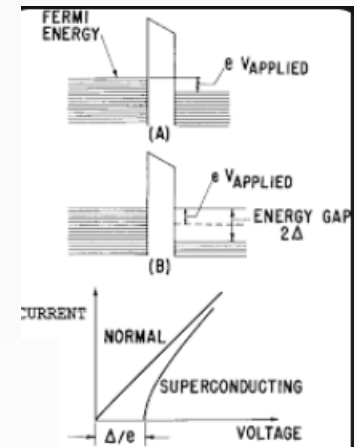
Artificial intelligence advice:

The temperature dependence of the energy gap ($\Delta(T)$) in superconductors over the entire temperature range can be described by a more precise formula derived from the BCS (Bardeen-Cooper-Schrieffer) theory. The formula is:

$$\Delta(T) = \Delta(0) \tanh \left(\frac{\pi k_B T_c}{\Delta(0)} \sqrt{\frac{T_c}{T} - 1} \right)$$

where:

- $\Delta(0)$ is the energy gap at absolute zero temperature,
- T is the temperature,
- T_c is the critical temperature,
- k_B is the Boltzmann constant.

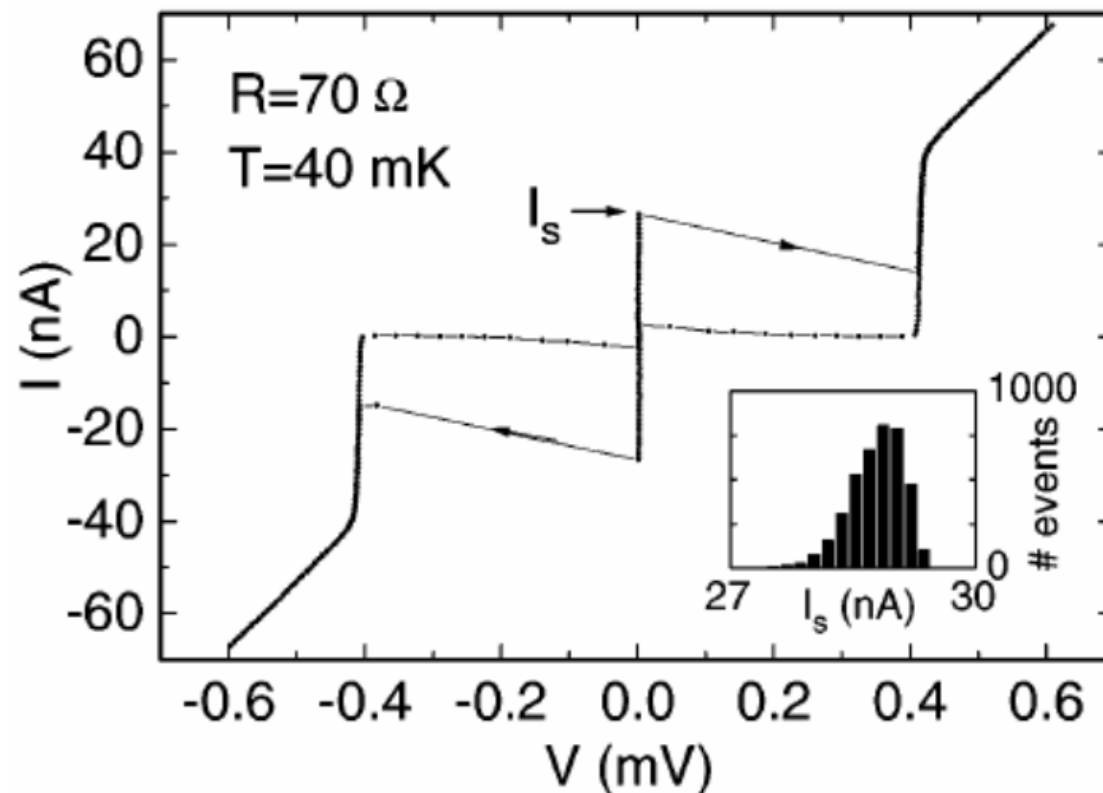


If the oxide barrier in the tunnel junction is sufficiently thin then one gets Josephson junction, which has zero resistance

FIG 3 - uploaded by [Daniel Esteve](#)
Content may be subject to copyright.

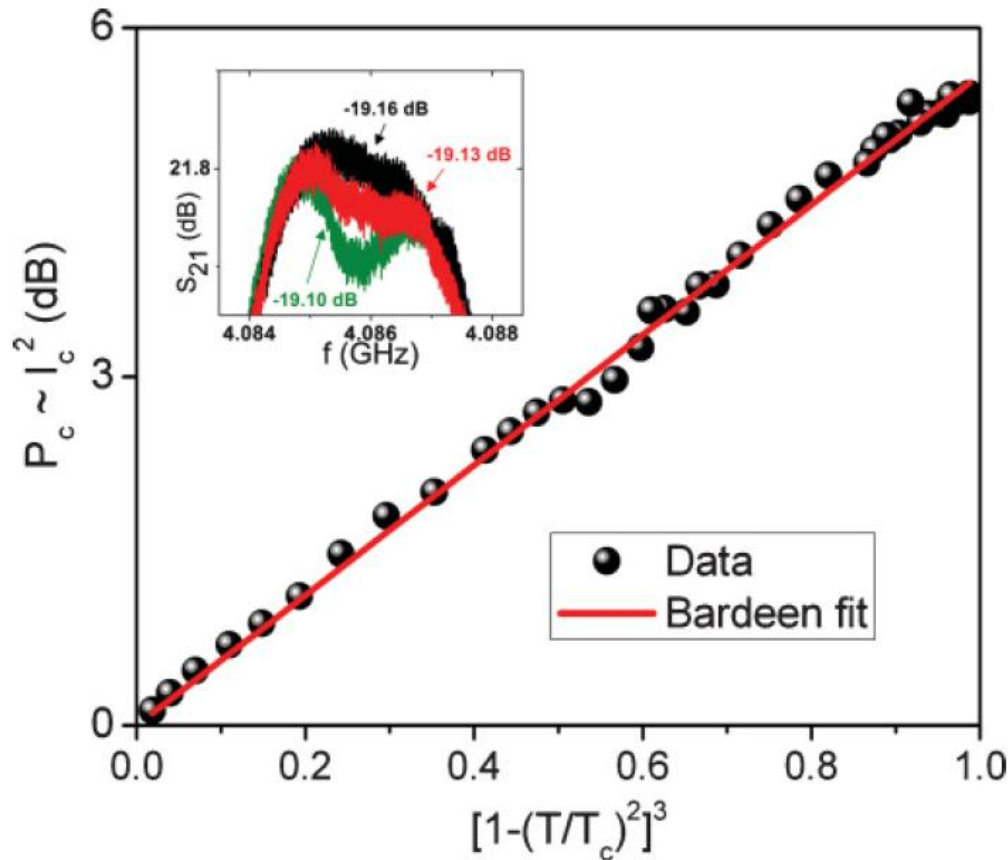
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Large scale I - V characteristic of a Josephson junction corresponding to the circuit of Fig. 2. The switching at current I_s from the diffusion branch (vertical branch in the center of the characteristic) to the quasiparticle branch of the junction is

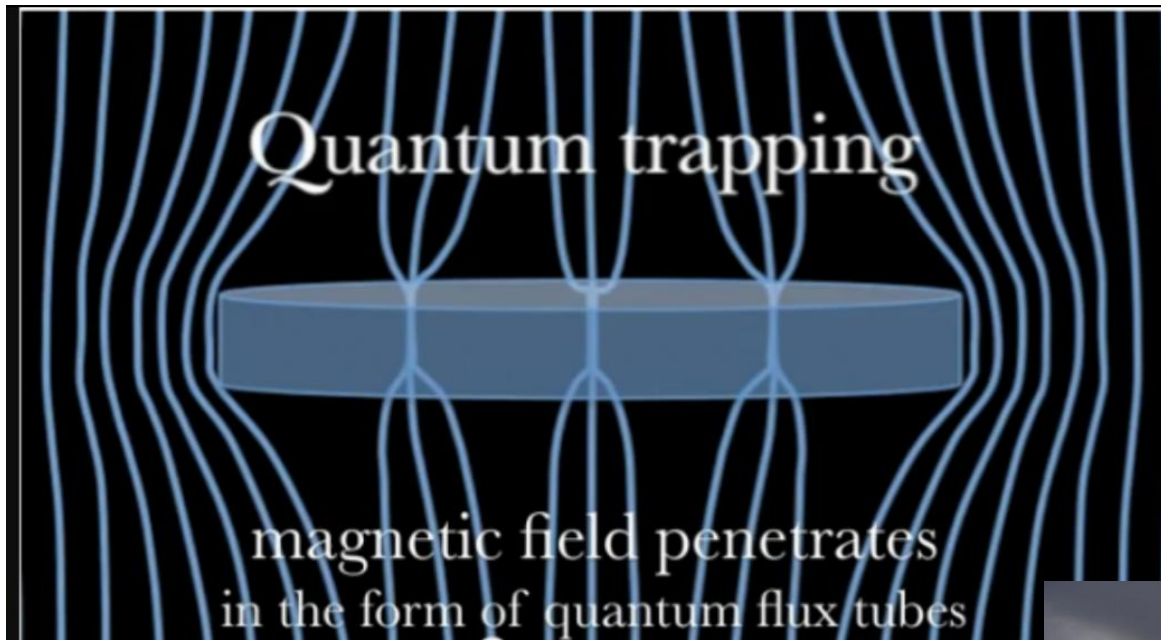
Critical current of a thin superconducting wire. Bardeen formula.



M. W. Brenner, S. Gopalakrishnan, J. Ku, T. J. McArdle, J. N. Eckstein, N. Shah, P. M. Goldbart, and Alexey Bezryadin, "Cratered Lorentzian response of driven microwave superconducting nanowire-bridged resonators: Oscillatory and magnetic-field induced stochastic states", PHYSICAL REVIEW B 83, 184503 (2011)

$$I_c = I_c(0)[1 - (T/T_c)^2]^{3/2}$$

Magnetic field effect: Superconducting vortices

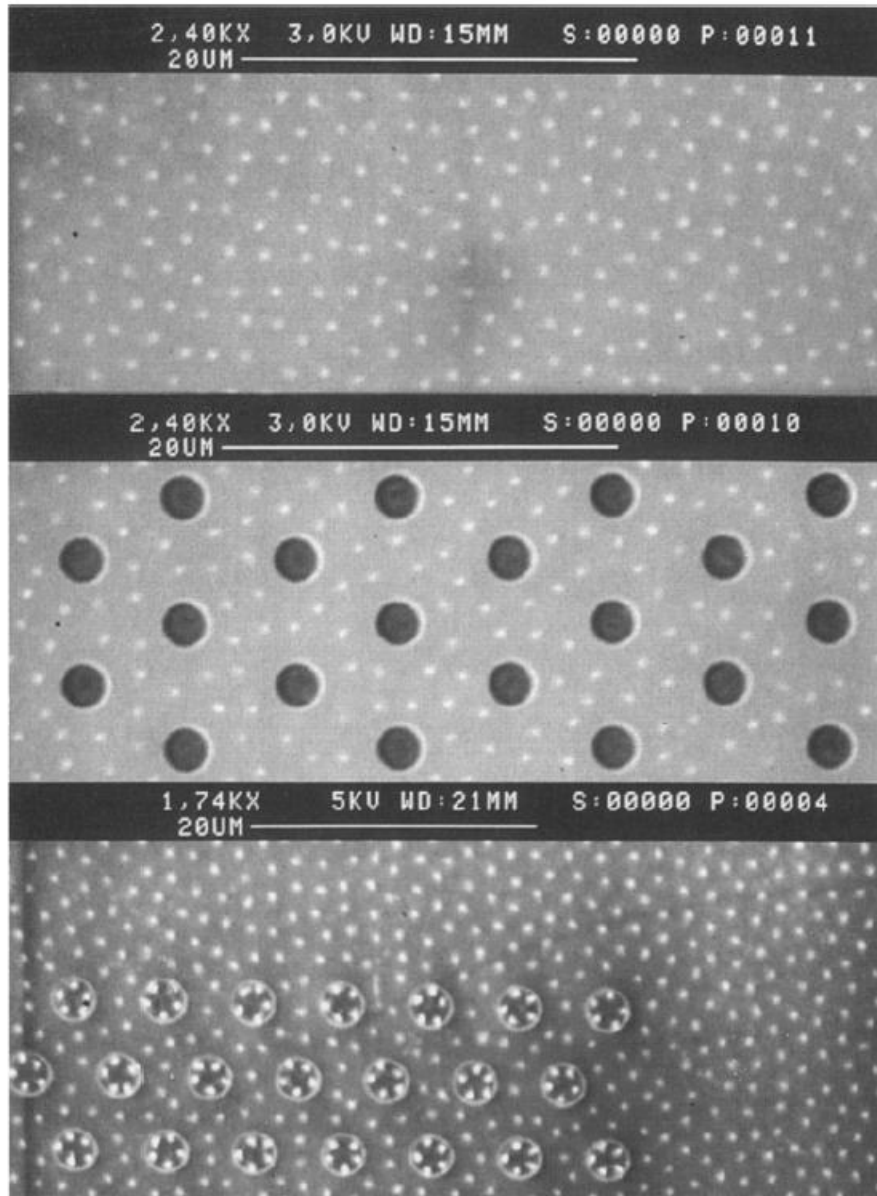


In superconductivity, a fluxon (also called an **Abrikosov vortex** or **quantum vortex**, **fluxons**) is a vortex of supercurrent in a type-II superconductors

<https://blog.tmcnet.com/blog/tom-keating/technology-and-science/quantum-levitation-back-to-the-future-hoverboard.asp>



Vortices in superconducting films with “through” and “blind” holes (“antidots”)



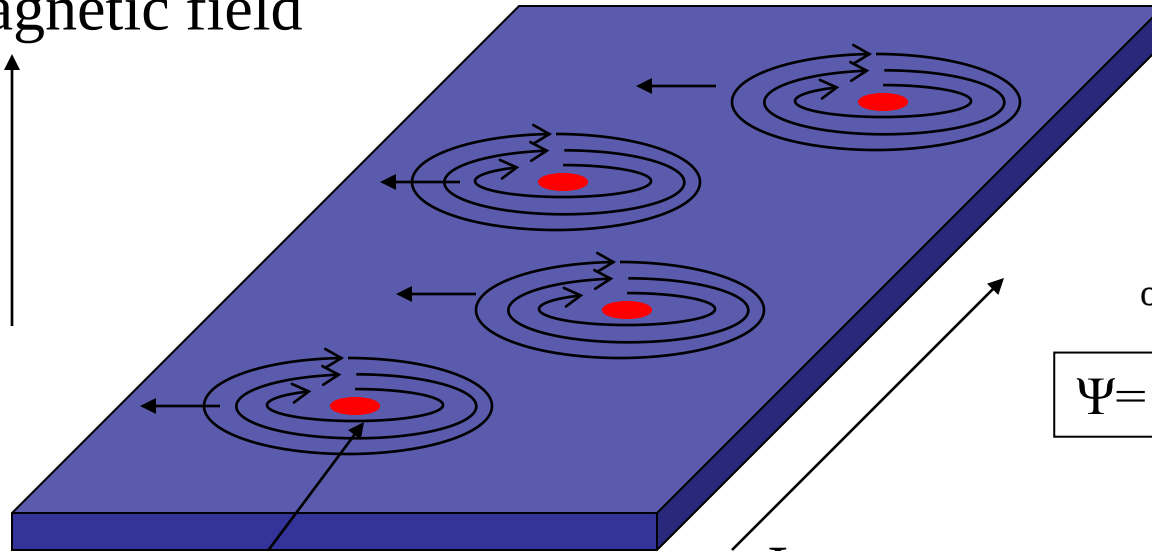
A. Bezryadin and B. Pannetier
“Role of Edge Superconducting States in
Trapping of Multi-Quanta Vortices by
Microholes. Application of the Bitter
Decoration Technique”,
J. of Low Temp. Phys., V.102, p.73 (1996).

Vortices are quantized tubes carrying magnetic field into superconductor

Magnetic field creates vortices--

Vortices cause dissipation (i.e. a non-zero electrical resistance), if they move

B -magnetic field



Wave function
of all superconducting electrons:

$$\Psi = |\Psi| \exp(i\phi) = |\Psi| \exp(i\theta)$$

amplitude

phase

Vortex core (red): normal, not superconducting; diameter $\xi \sim 10$ nm

The current is extended to a scale λ , which is larger than ξ in type II superconductors (such as thin films of any material)

Reminder: single electron in empty space

Wave function: $\Psi = |\Psi| \exp(ikx) = |\Psi| [\cos(kx) + i \sin(kx)]$

Wave number: $k = 2\pi/\lambda$

$$i*i = -1$$

General form: $\Psi = |\Psi| \exp(-i\phi)$

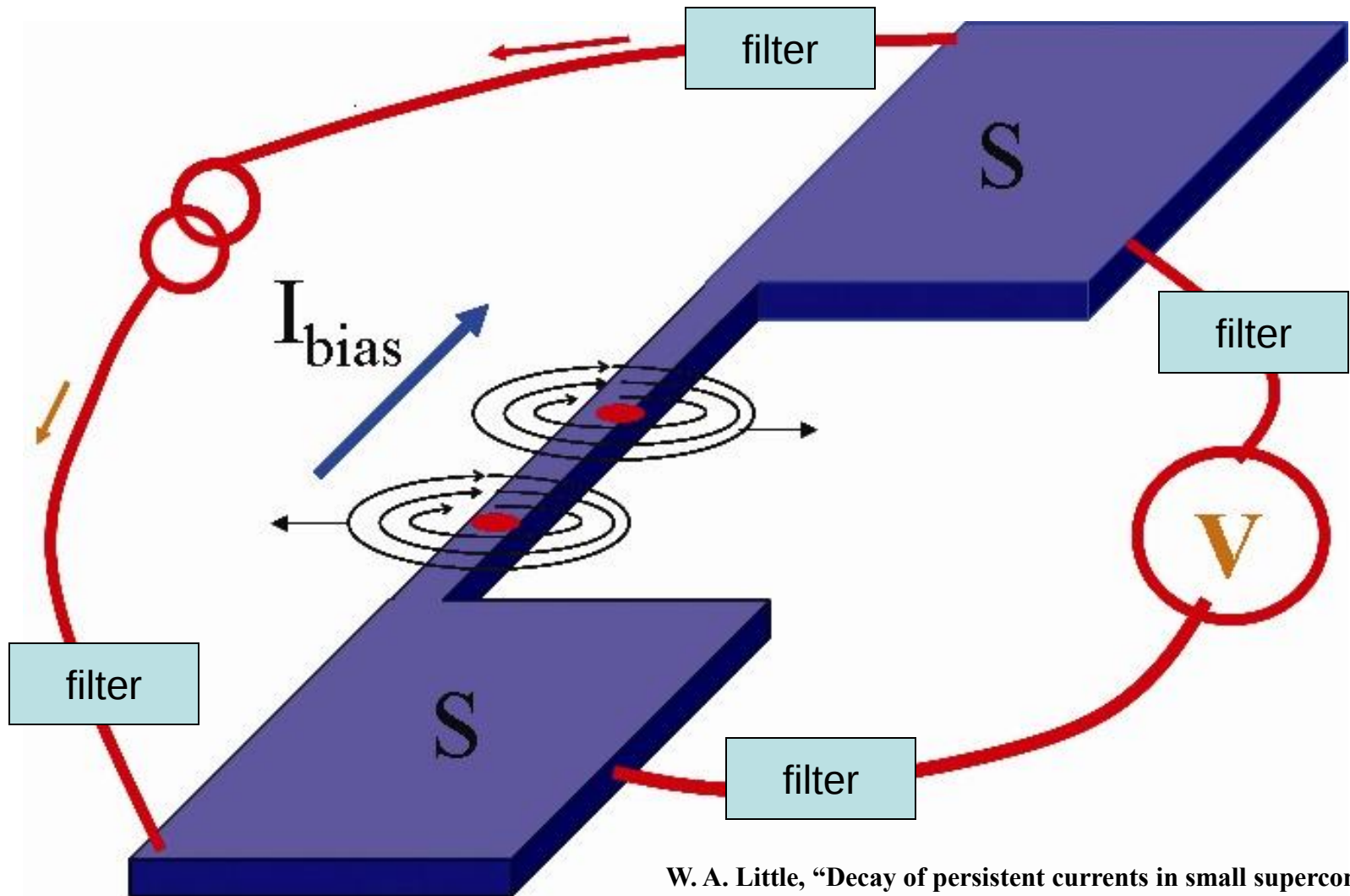
In this example of a plane wave, the phase is: $\phi = kx$

Momentum: $p = (\hbar/2\pi)k$

DC transport measurement schematic to detect passing vortices

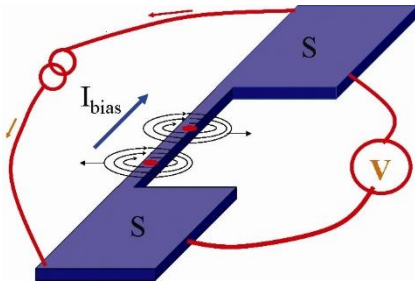
Bleu: superconducting film and wire

Red: Phase slip events or crossing vortices



W. A. Little, "Decay of persistent currents in small superconductors",
Physical Review, V.156, pp.396-403 (1967).

How to use voltage to determine the rate of phase slips?



Key principle: every time a vortex crosses the wire the phase difference changes by 2π .

Phase evolution equation: $d\phi/dt = 2eV/\hbar$

Simplified derivation:

1. Time-dependent Schrödinger equation with fixed energy:

$$i\hbar(d\Psi/dt) = E \Psi$$

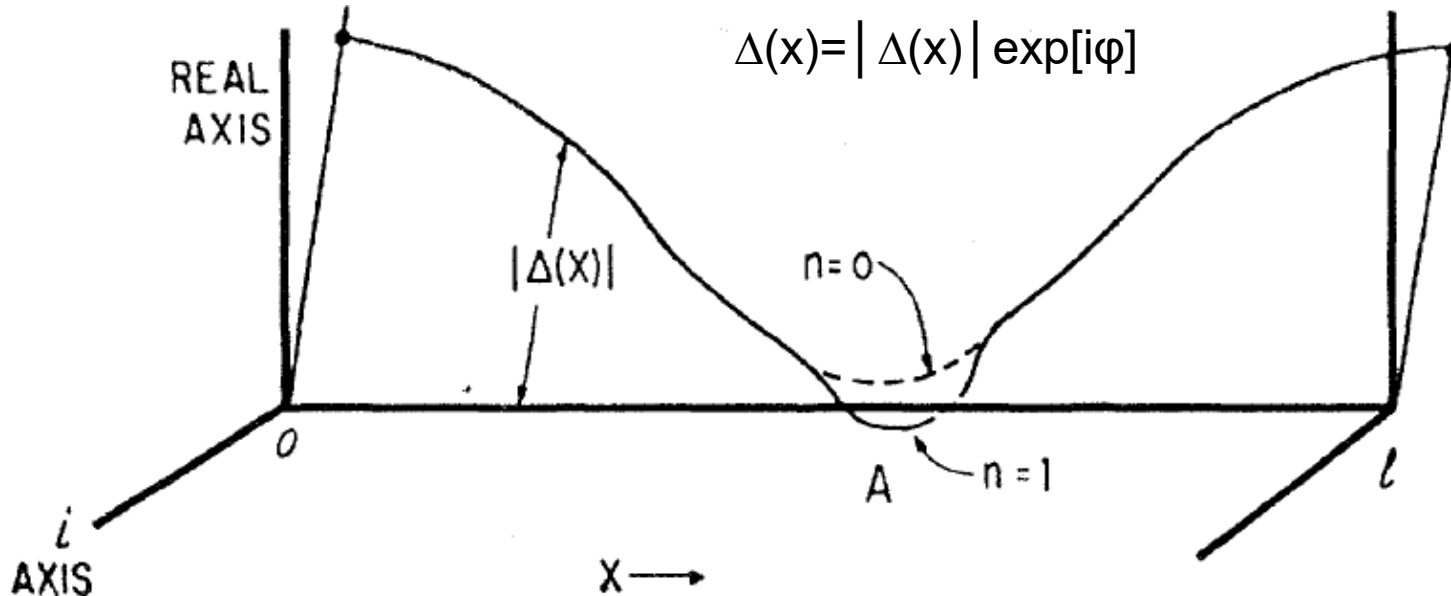
2. The solution is: $\Psi = \exp(-iEt/\hbar)$ (here E is the energy)

3. The phase of the wavefunction is $\phi = Et/\hbar$

4. The energy is defined by the electric potential (voltage), V as follows: $E = 2eV$. Note that the effective charge of superconducting electrons is $2e$, where “ e ” is the charge of one electron. Such superconducting electron pairs are called Cooper pairs.

Thus, the resulting equation is: $d\phi/dt = 2eV/\hbar$

Thin superconducting wire have some nonzero electrical resistance due to **Little's Phase Slips**

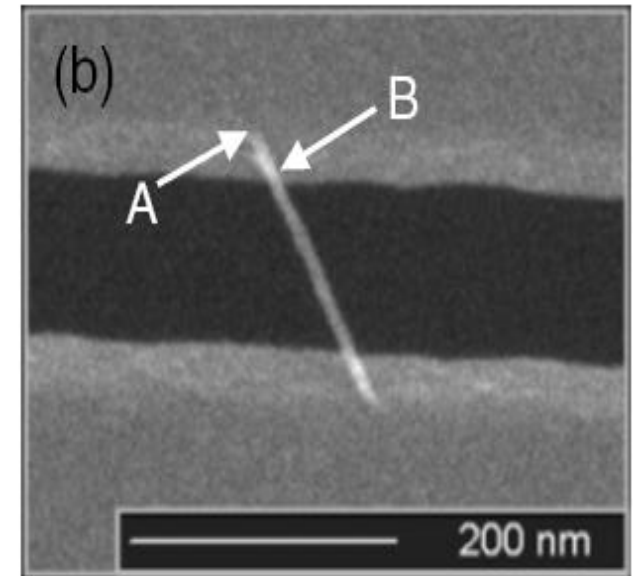
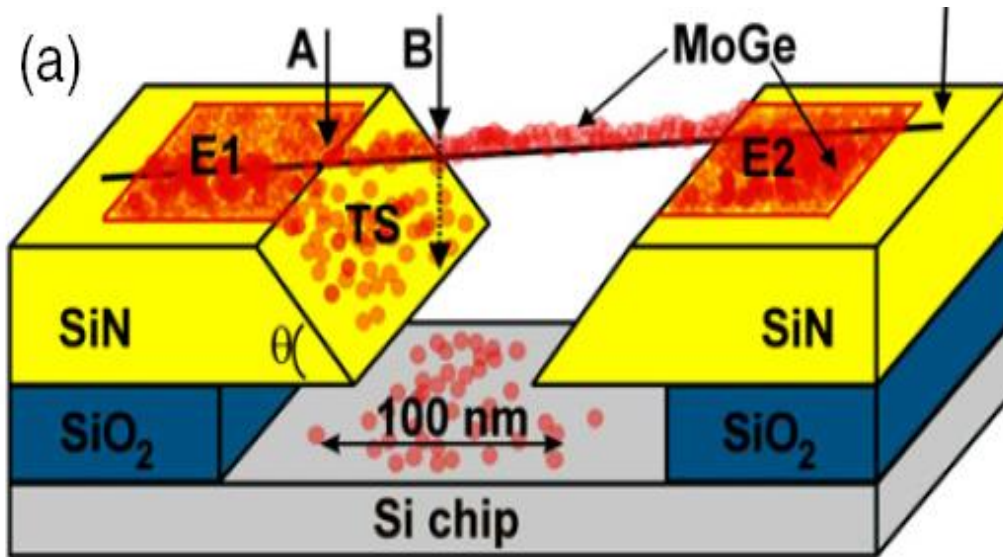


**W. A. Little, “Decay of persistent currents in small superconductors”,
Physical Review, V.156, pp.396-403 (1967).**

Two types of phase slips (PS) can occur:

1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

Method of Molecular Templating



Si/ SiO₂/SiN substrate with undercut

~ 0.5 mm Si wafer

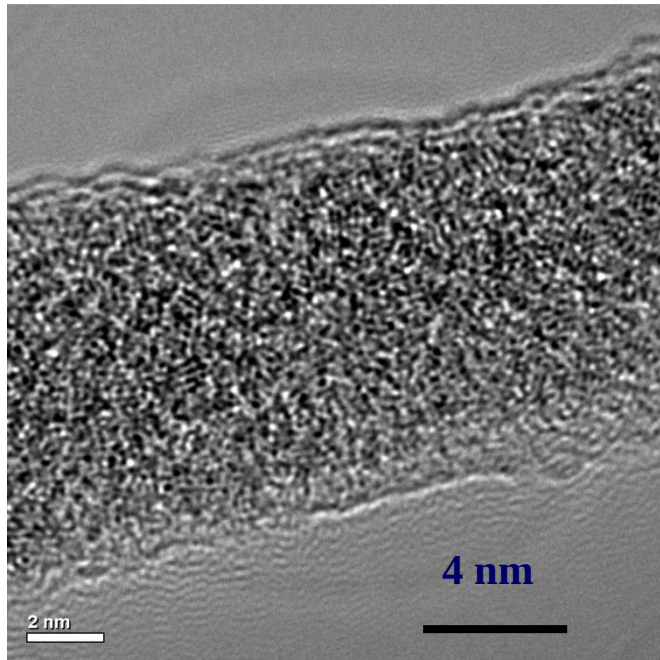
500 nm SiO₂

60 nm SiN

Width of the trenches $\sim 50 - 500$ nm

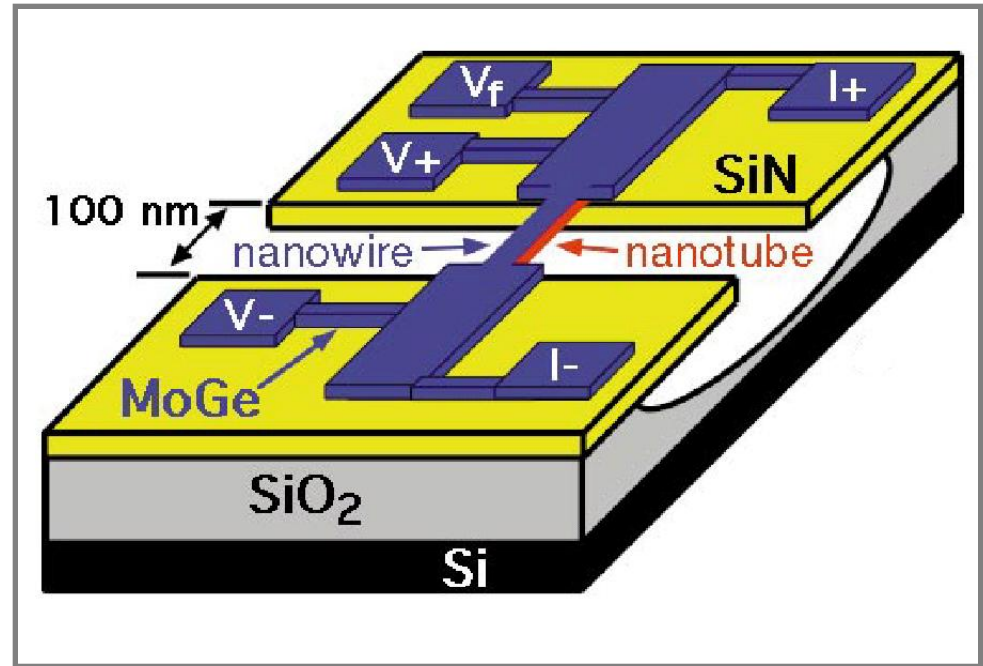
**HF wet etch for ~10 seconds
to form undercut**

Sample Fabrication



TEM image of a wire shows amorphous wire morphology.

Nominal MoGe thickness = 3 nm



Schematic picture of the pattern
Nanowire + Film Electrodes used in
transport measurements



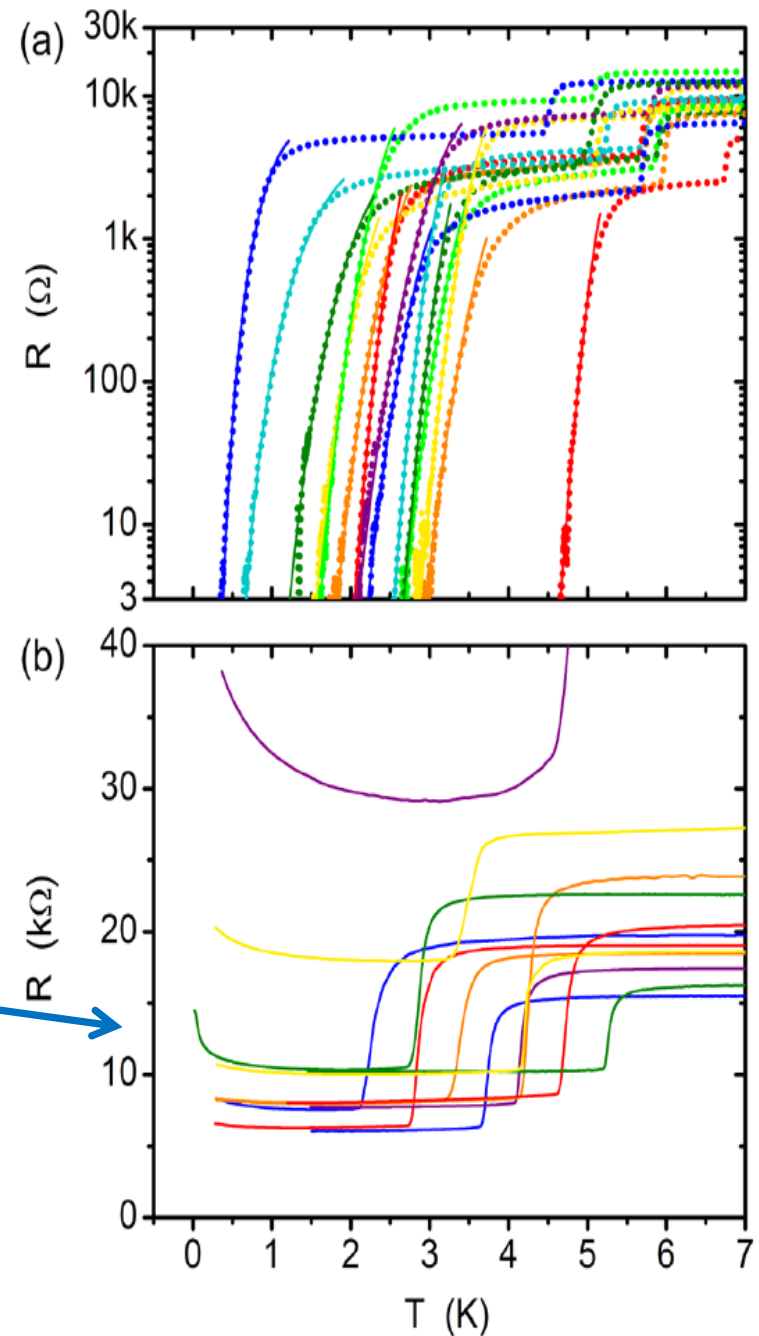
Dichotomy in nanowires: Evidence for superconductor- insulator transition (SIT)

$$R=V/I \quad I \sim 3 \text{ nA}$$

The difference between samples is the amount of the deposited Mo79Ge21.

Thin wires become insulating if their normal resistance is larger than resistance quantum $h/4e^2 = 6.5 \text{ k}\Omega$

The insulating behavior is due to proliferation of quantum phase slips

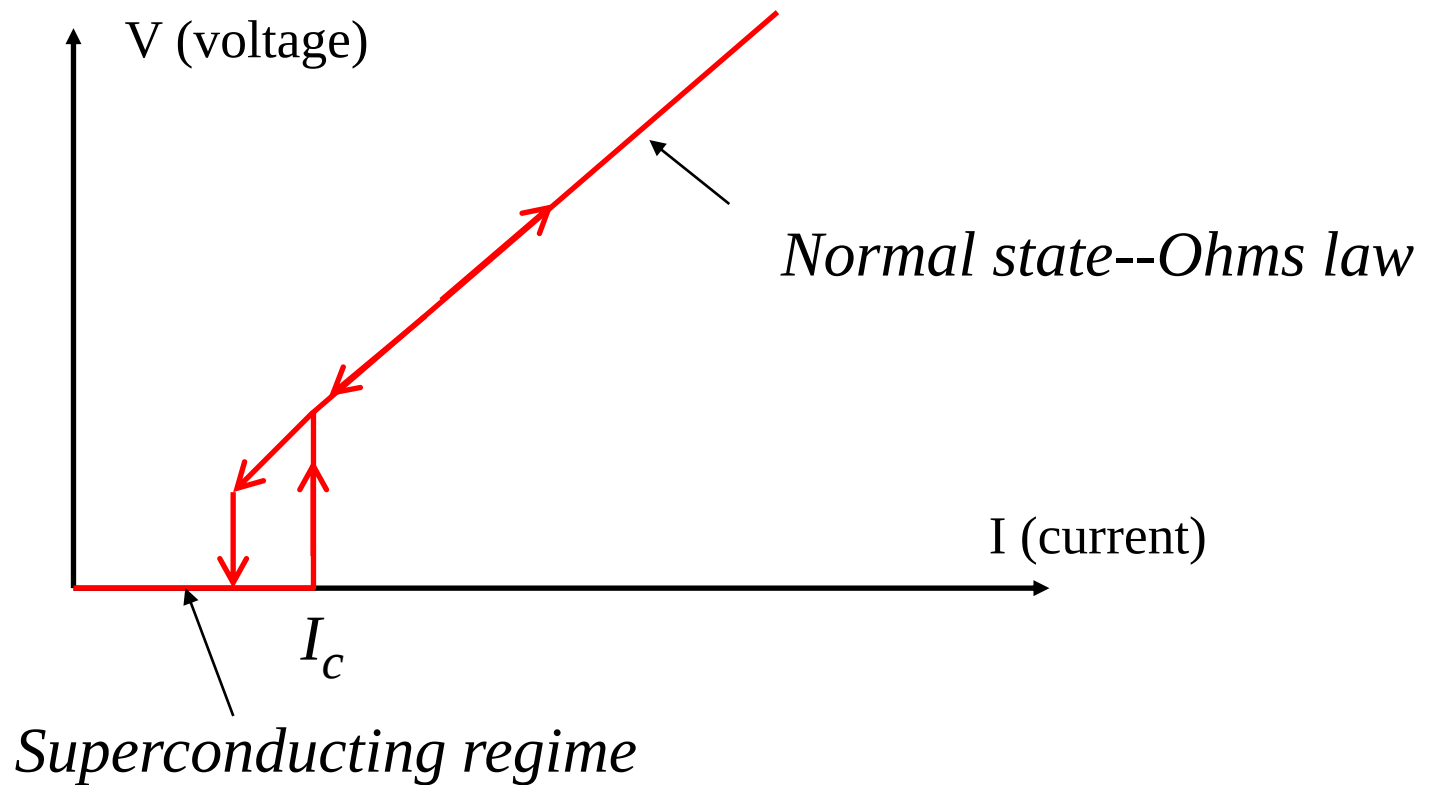


Bollinger, Dinsmore, Rogachev, Bezryadin,
Phys. Rev. Lett. **101**, 227003 (2008)



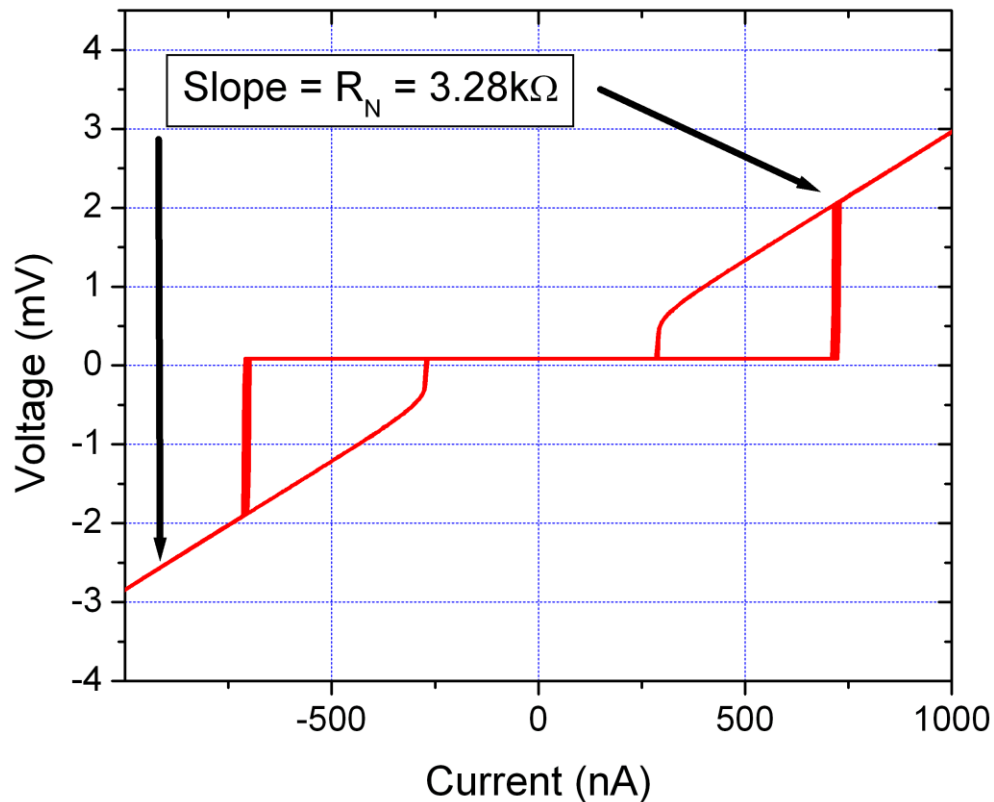
Expected voltage-current curve

Electrical resistance is zero only if current is not too strong

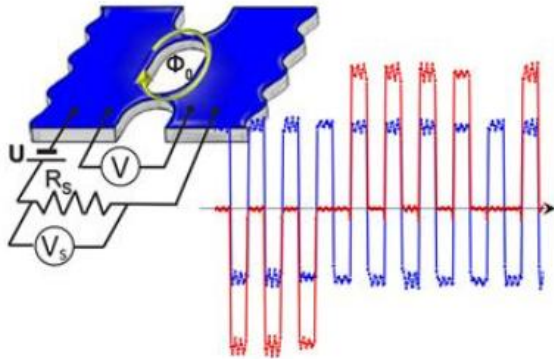


Experimental voltage-current curve.

Fluctuations of the switching current are due to Little's phase slips



Superconducting nanowire memory

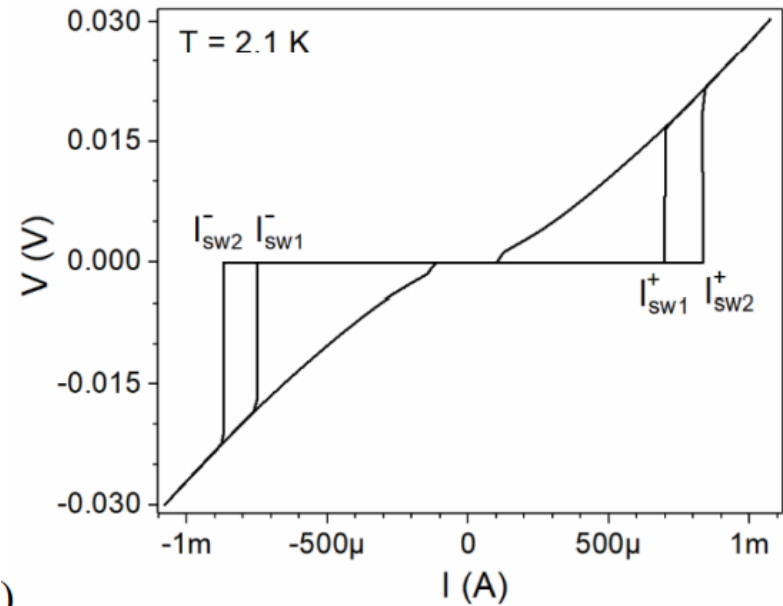
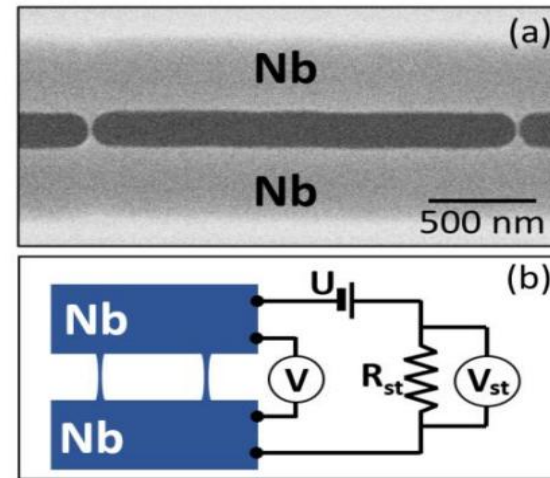


Volume 118, Issue 11, 15 Mar. 2021

Supercurrent-controlled kinetic inductance superconducting memory element

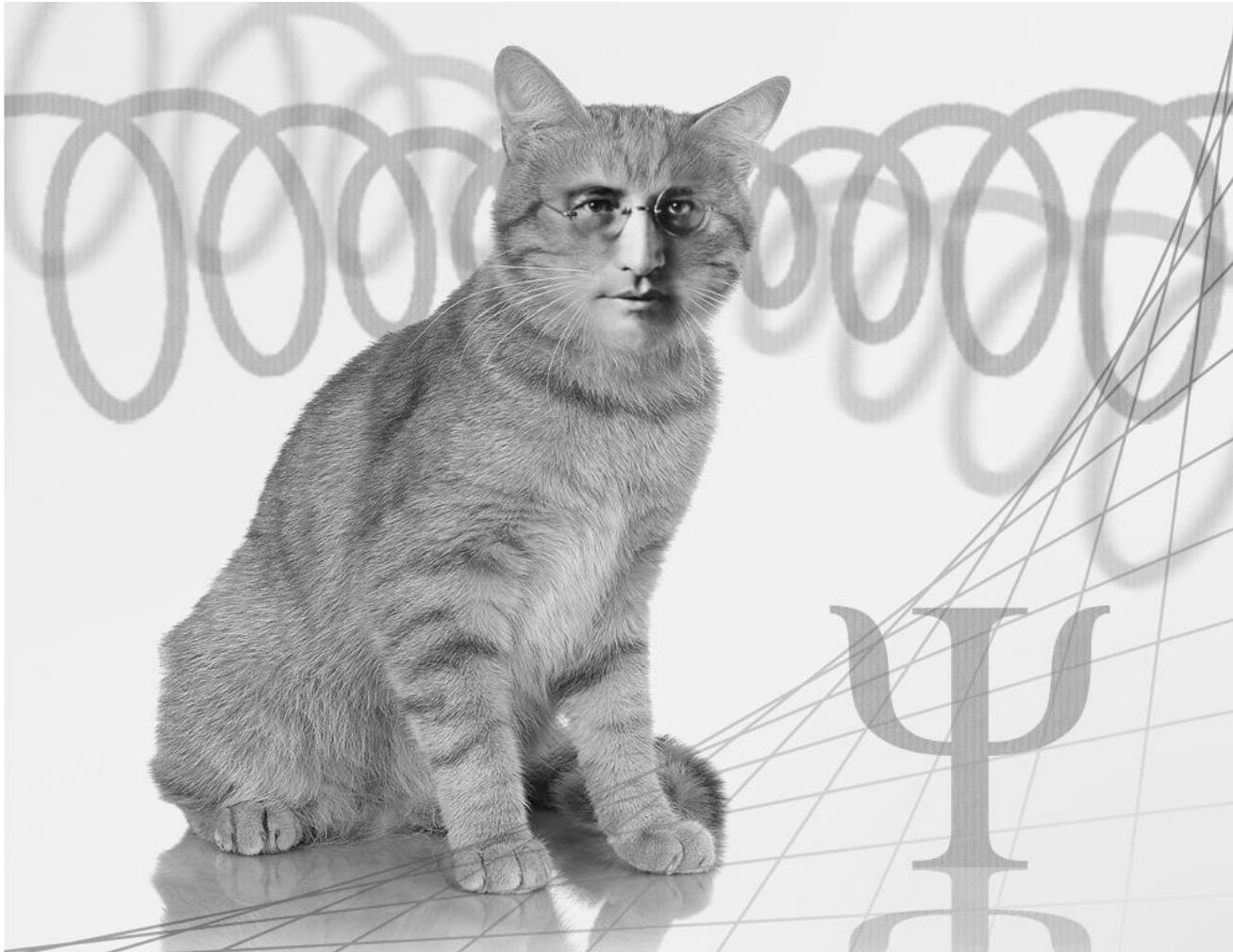
Appl. Phys. Lett. 118, 112603 (2021); doi: 10.1063/5.0040563

Eduard Ilin, Xiangyu Song, Irina Burkova, Andrew Silge, Ziang Guo, Konstantin Ilin, and Alexey Bezryadin

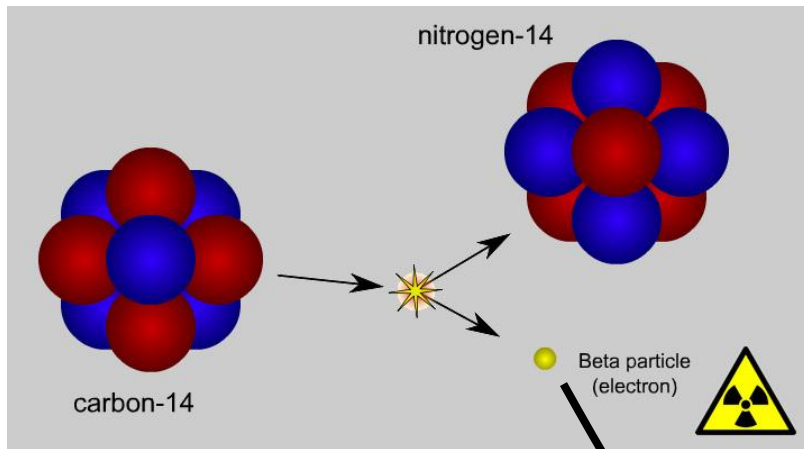


Schrödinger cat – the ultimate macroscopic quantum phenomenon

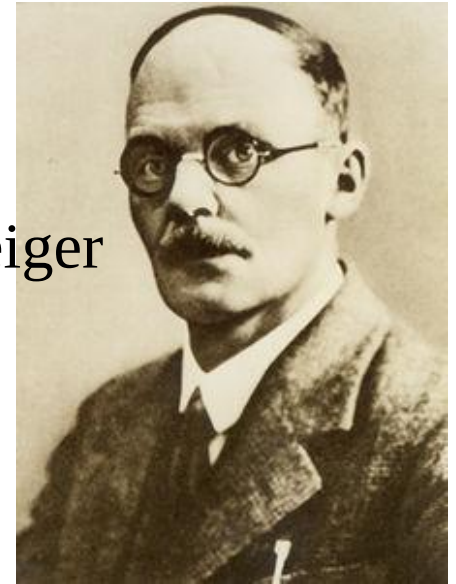
E. Schrödinger, Naturwiss. **23** (1935), 807.



Schrödinger cat – thought experiment



Hans Geiger



Geiger counter





Linearity of the Schrödinger's equation

Suppose Ψ_1 is a valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_1}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial x^2} + U(x)\psi_1$$

And suppose that Ψ_2 is another valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_2}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_2}{\partial x^2} + U(x)\psi_2$$

Then $(\Psi_1 + \Psi_2)/\sqrt{2}$ is also a valid solution, because:

$$i\hbar \frac{\partial (\psi_1 + \psi_2)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} + U(x)(\psi_1 + \psi_2)$$

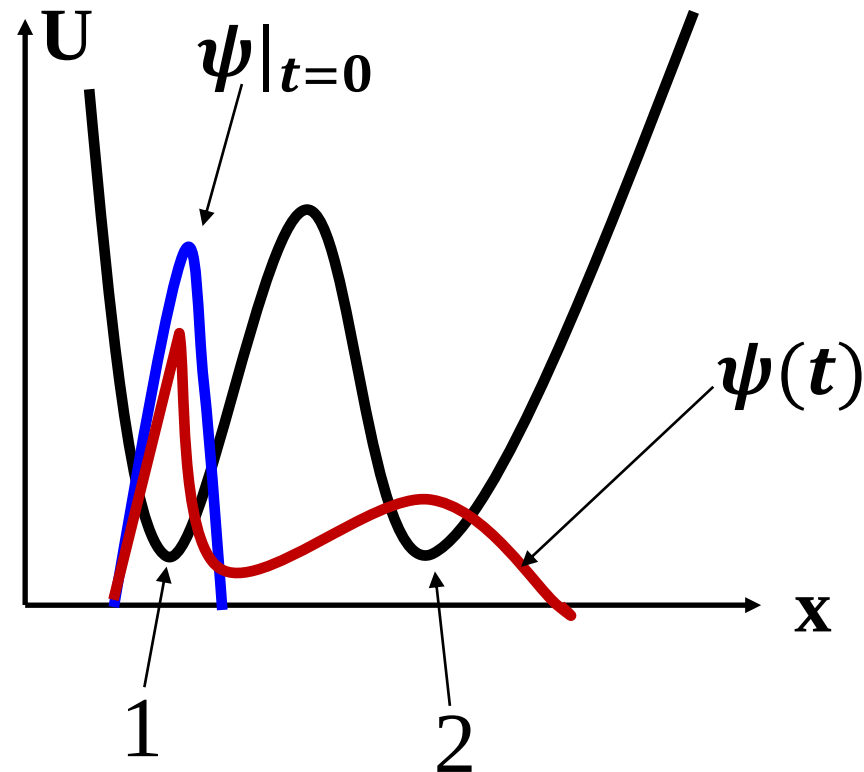
The state $(\Psi_1 + \Psi_2)/\sqrt{2}$ is a new combined state which is called “quantum superposition” of state (1) and (2)



George Gamow

(He also helped to
developed
Big Bang theory)

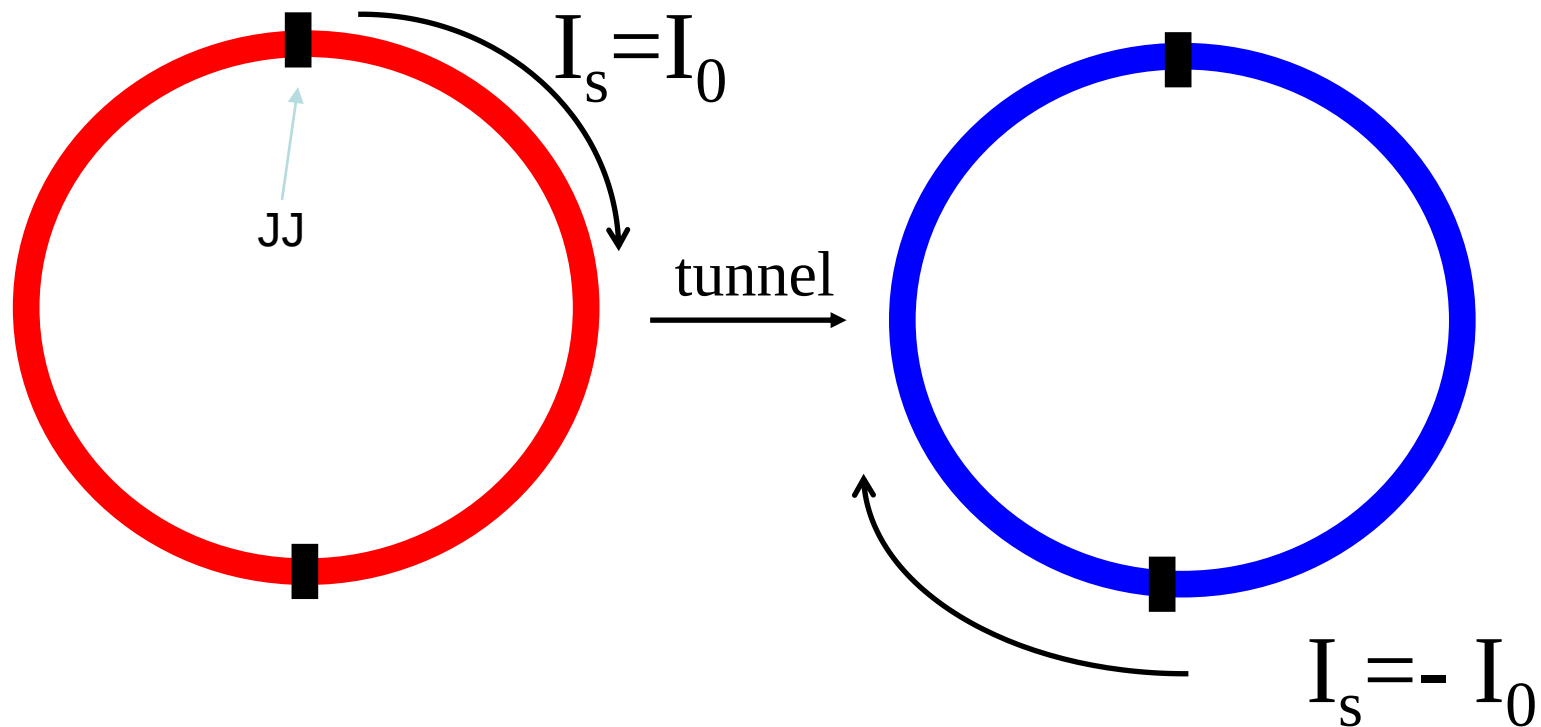
Quantum tunneling



**Quantum tunneling is possible
since quantum superpositions of
states are possible.**

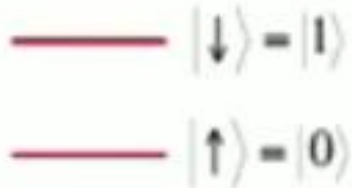


What sort of tunneling we will consider?



- Red color represents some strong current in the superconducting wire loop
- Blue color represents zero current in the loop

Types of Qubit



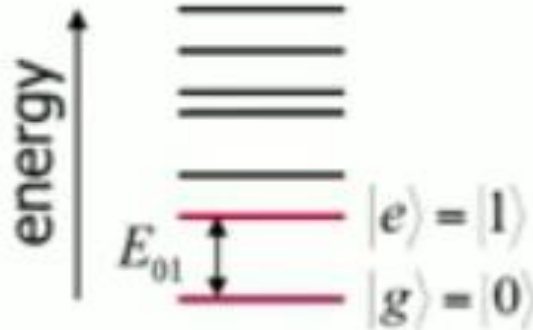
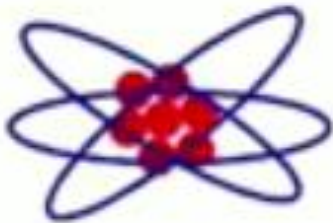
single spin-1/2

Quantum state:

$$|\psi\rangle = A^*|0\rangle + B^*|1\rangle$$

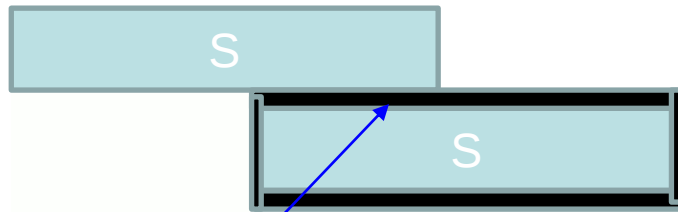
$$A^2 + B^2 = 1$$

A and B are
complex numbers

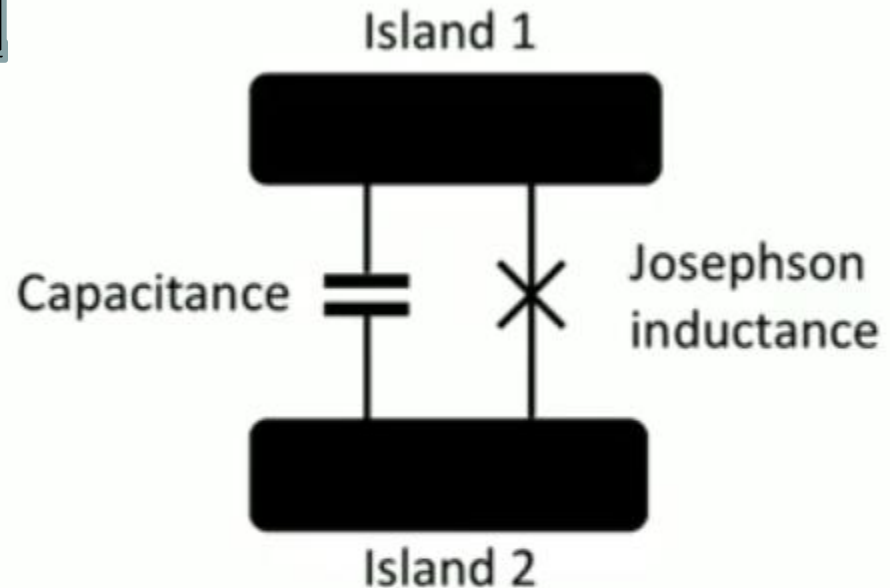


single atom

Transmon Qubit



Electrical schematic of qubit

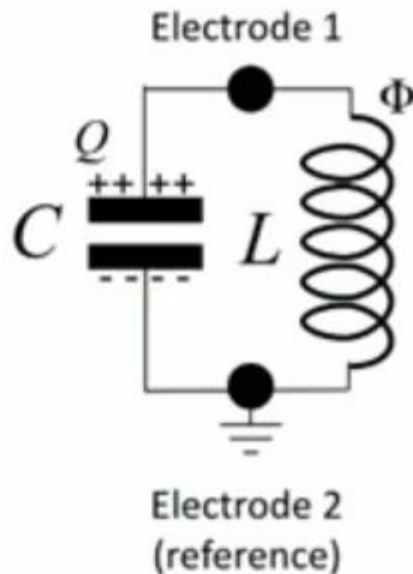


Theory of the transmon: J. Koch *et al.*, Phys Rev. A **76**, 042319 (2007)

Theory of transmons: J. Koch et al., Phys. Rev. A **76**, 042319 (2007).

Quantization of electrical circuits

The quantized LC oscillator



Hamiltonian:

$$\hat{H}_{LC} = \underbrace{\frac{\hat{Q}^2}{2C}}_{\text{Capacitive term}} + \underbrace{\frac{\hat{\Phi}^2}{2L}}_{\text{Inductive term}}$$

Canonically conjugate variables:

$\hat{\Phi}$ = Flux through the inductor.

$\hat{Q}$ = Charge on capacitor plate.

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

Discrete energy spectrum of the LC-circuit

Correspondence with simple harmonic oscillator

$$\hat{H}_{\text{LC}} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}$$

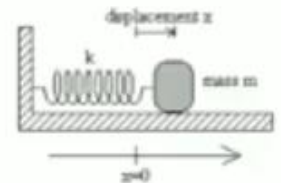
$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

$$\hat{H}_{\text{SHO}} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m}$$

$$[\hat{X}, \hat{P}] = i\hbar$$

Correspondence:

$$\begin{array}{ll} \hat{\Phi} \leftrightarrow \hat{X} & L \leftrightarrow \frac{1}{k} \\ \hat{Q} \leftrightarrow \hat{P} & C \leftrightarrow m \end{array} \quad \omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}$$



Solve using ladder operators:

$$\hat{a} = \left(\frac{\hat{Q}}{Q_{\text{zpf}}} - i \frac{\hat{\Phi}}{\Phi_{\text{zpf}}} \right)$$

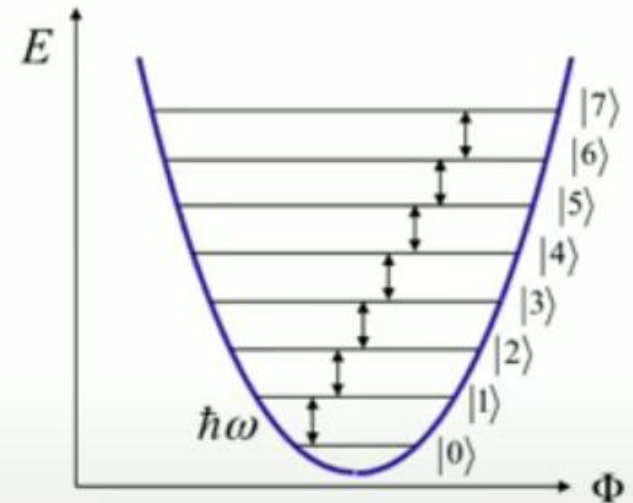
$$\Phi_{\text{zpf}} = \sqrt{2\hbar Z}$$

$$Q_{\text{zpf}} = \sqrt{2\hbar / Z}$$

$$\hat{a}^\dagger = \left(\frac{\hat{Q}}{Q_{\text{zpf}}} + i \frac{\hat{\Phi}}{\Phi_{\text{zpf}}} \right)$$

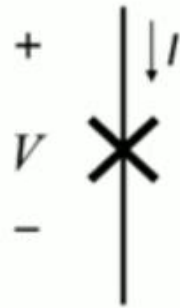
$$Z = \omega L = \frac{1}{\omega C} = \sqrt{\frac{L}{C}}$$

$$\hat{H}_{\text{LC}} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \quad [\hat{a}, \hat{a}^\dagger] = 1$$



Non-harmonicity is the key factor

The Josephson junction



$$I = I_c \sin\left(2\pi \frac{\Phi}{\Phi_0}\right)$$

$$V = \frac{\hbar}{2e} \frac{d\phi}{dt}$$

$$\Phi_0 = \frac{h}{2e}$$

flux quantum

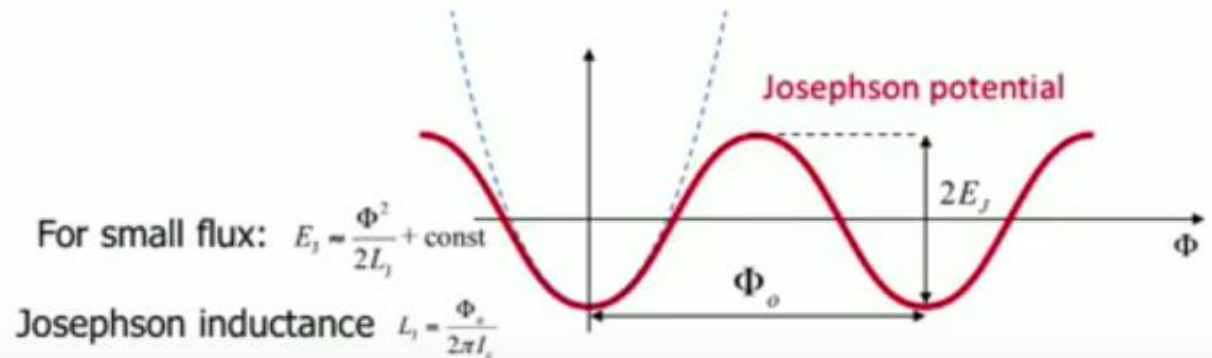


S superconductor-
I insulator-
S superconductor
tunnel junction

$$I_c = \frac{\pi \Delta}{2e R}$$

$$E_{\text{stored}} = E_J \left(1 - \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)\right)$$

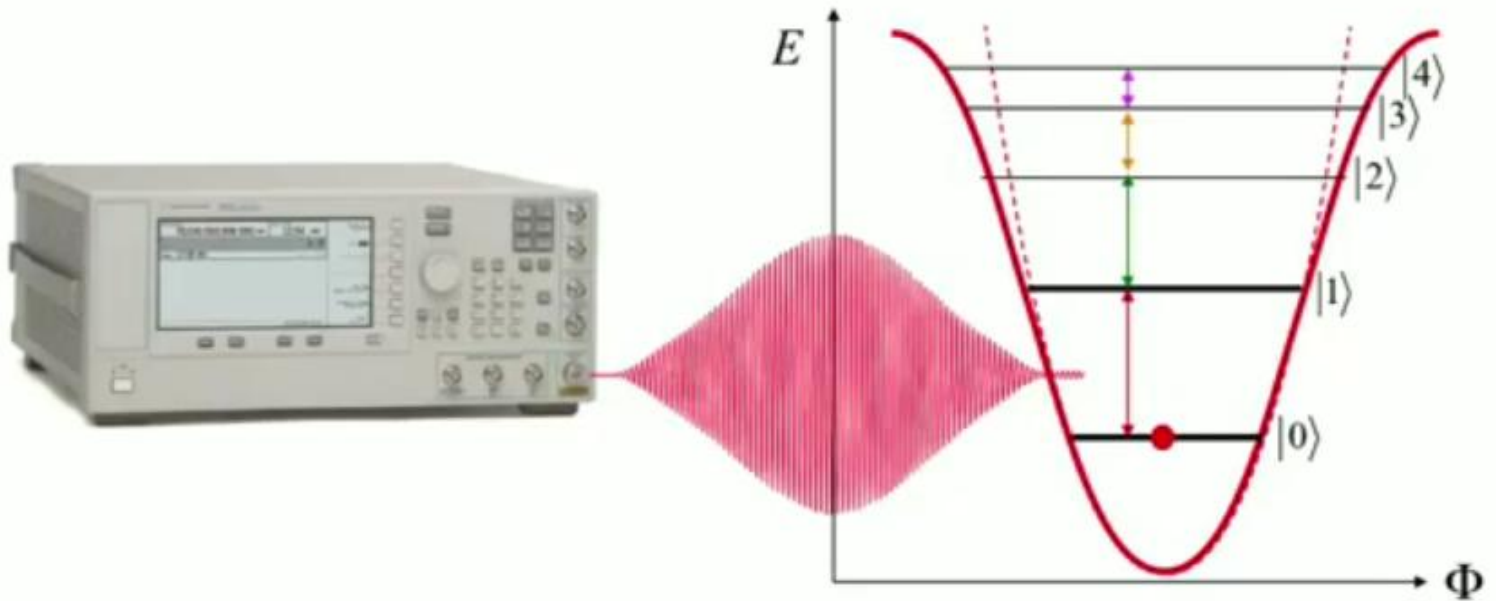
$$E_J = \frac{I_c \Phi_0}{2\pi} \quad \text{Josephson Energy}$$



M. Devoret, Les Houches Session LXIII (1995)

Non-harmonicity is the key factor

Transmon energy spectrum



Meissneron-transmon qubit

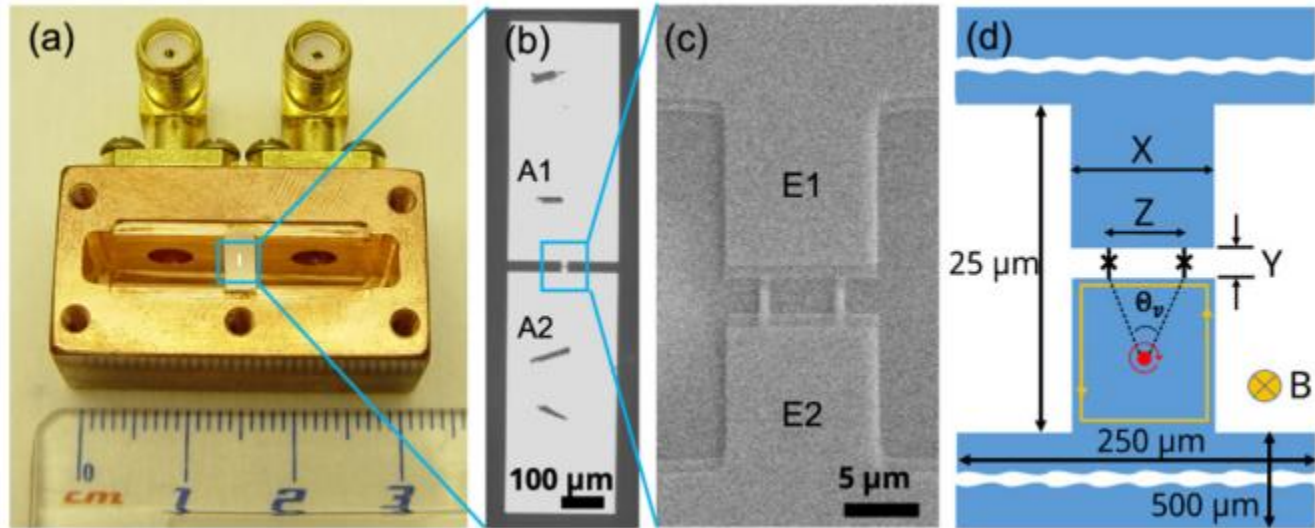


FIG. 1. (a) Optical image of the Meissner transmon qubit fabricated on a sapphire chip, which is mounted in the copper cavity. (b) A zoomed-in optical image of the qubit. Two rectangular pads marked A1 and A2 act as an RF antenna and shunt capacitor. (c) Scanning electron microscope (SEM) image of the electrodes marked E1 and E2, and a pair of JJs. (d) Schematics of the Meissner qubit. The X, Y, and Z denote the width, the distance between the electrodes, and the distance between two JJs, which are indicated by $\times$ symbols. The red dot and circular arrow around it in the bottom electrode represent a vortex and vortex current flowing clockwise, respectively. Θ_v is a polar angle defined by two dashed lines connecting the vortex and two JJs. The orange rectangular loop on the boundary of the bottom electrode indicates the Meissner current circulating counterclockwise.

Meisnerron-transmon qubit

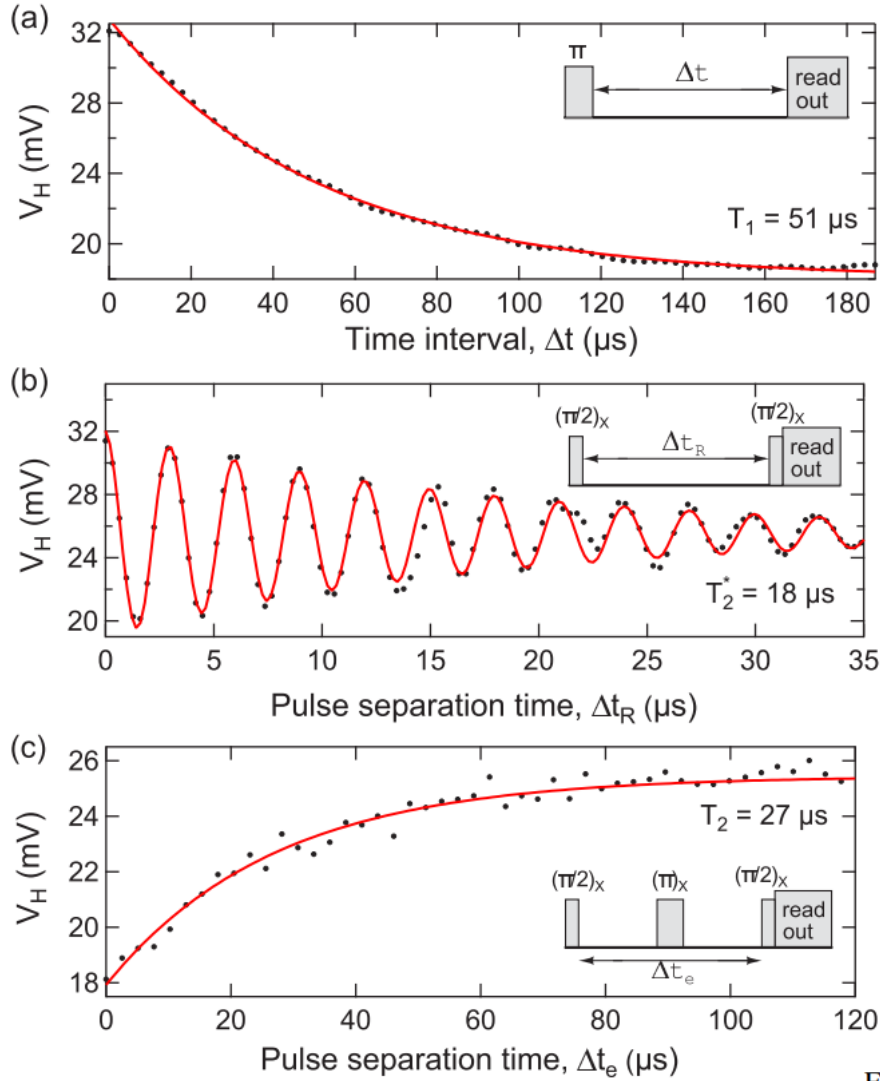


FIG. 4. Time domain measurements of the N7 sample at $B = 7.5$ mG. (a) Relaxation time measurement ($T_1 = 51 \mu s$). (b) Ramsey fringe experiment ($T_2^* = 18 \mu s$). (c) Hahn spin echo experiment ($T_2 = 27 \mu s$). The red solid lines are the fits to the data. See the main text for the fitting functions.

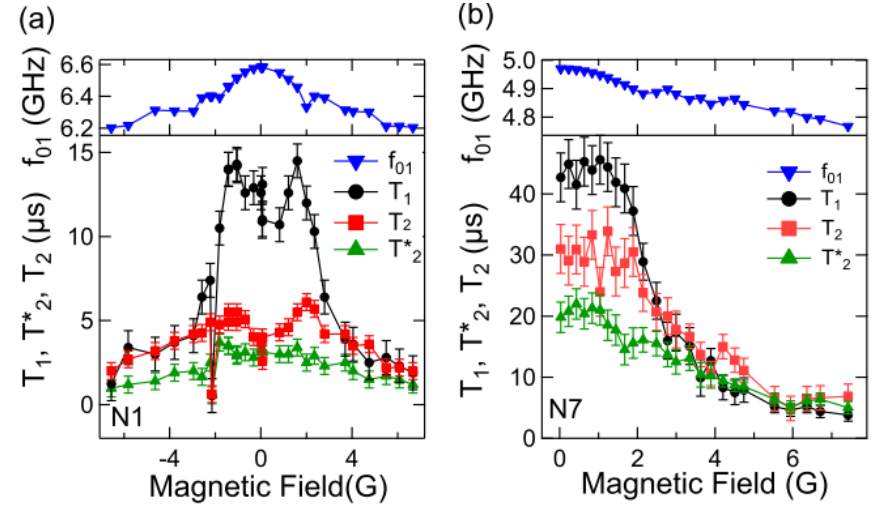


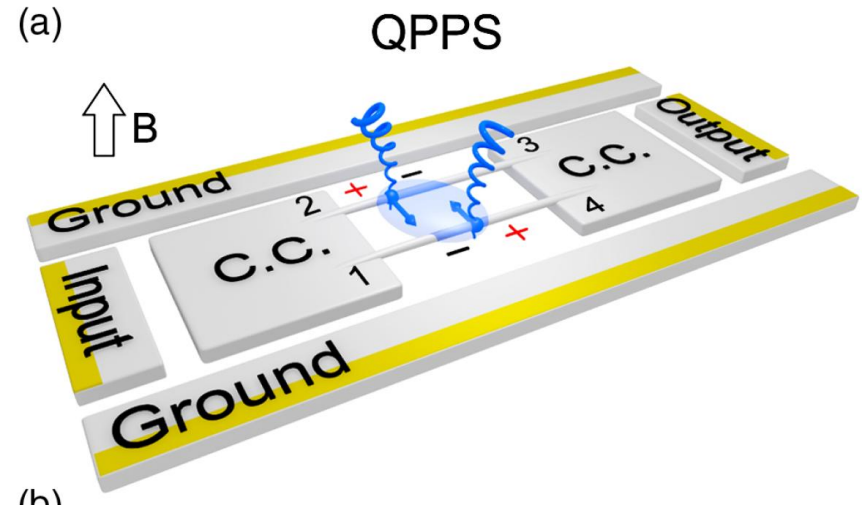
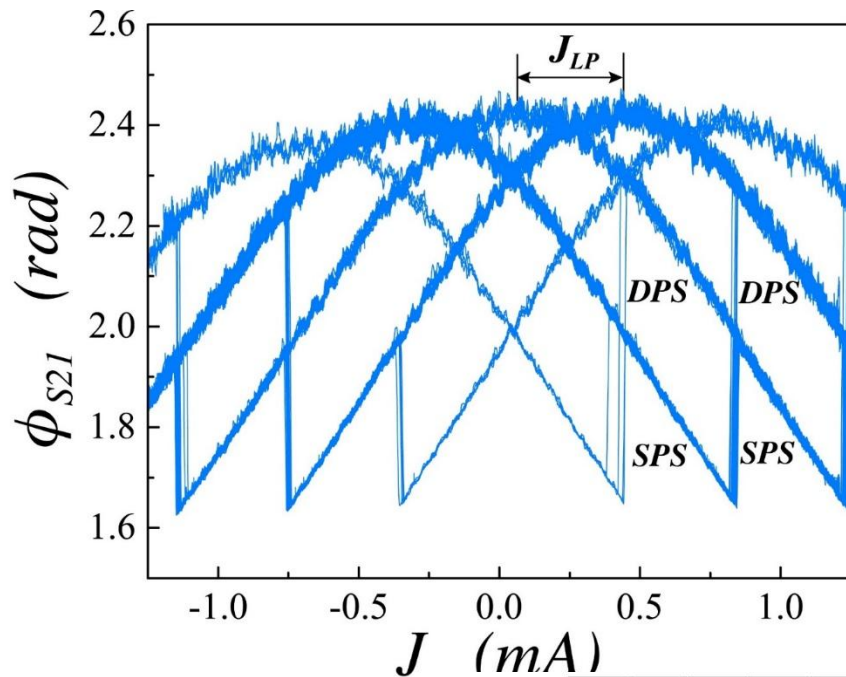
FIG. 6. The qubit transition frequencies (f_{01}) and three times scales (T_1 , T_2^* , and T_2) were measured at the sweet spots over the wide range of magnetic field for the N1 (a) and N7 (b).

Conclusions

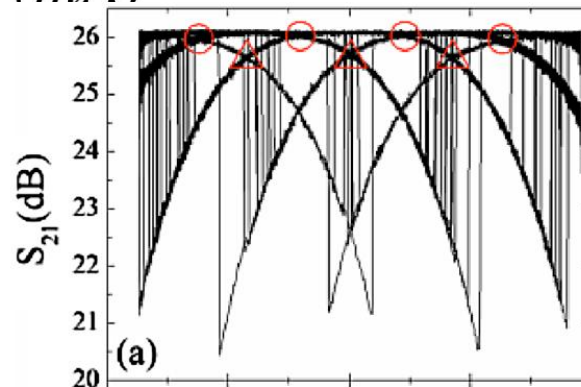
- Superconductivity is related to fundamental quantum phenomena. We have reviewed some of them. They will be discussed in more details in the future lectures.
- Superconductors have been used to create strong and stable magnetic fields, in levitating trains for example.
- Superconducting quantum interference devices enabled researches to measure very small magnetic fields, such as those produced by human brain.
- Superconductors are used to build qubits, which are the building blocks of quantum computers.



Superconducting nanowire memory: microwave readout



A. Belkin et al, PRX 5, 021023 (2015)

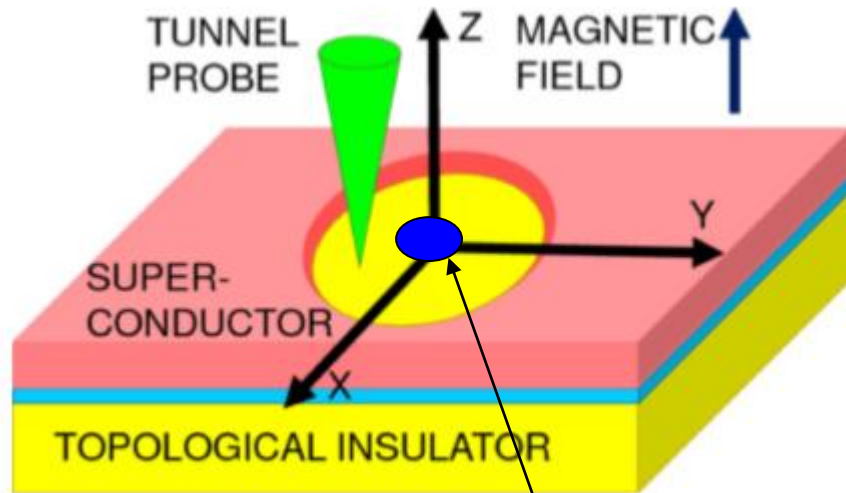


$T = 360$ mK
 $f = f_0(H=0)$

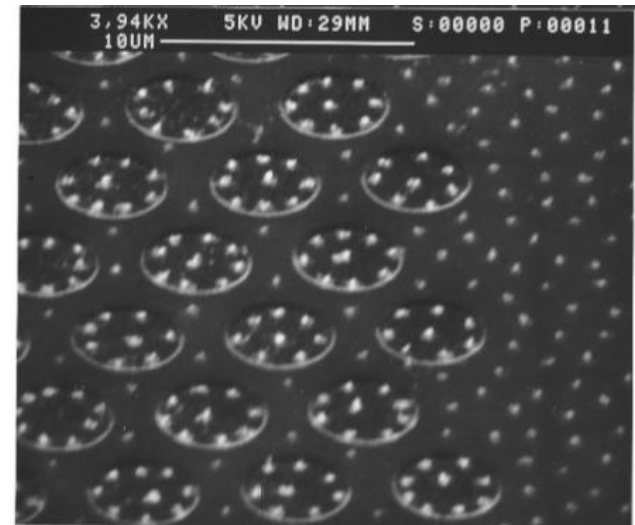
Andrey Belkin,^a Matthew Brenner,
 Thomas Aref, Jaseung Ku, and Alexey
 Bezryadin,
 PPLIED PHYSICS LETTERS **98**, 242504
 (2011)



Majorana modes in a vortex



Theory: Vortex in the nano-hole contains Majorana states (gap is about T_c)



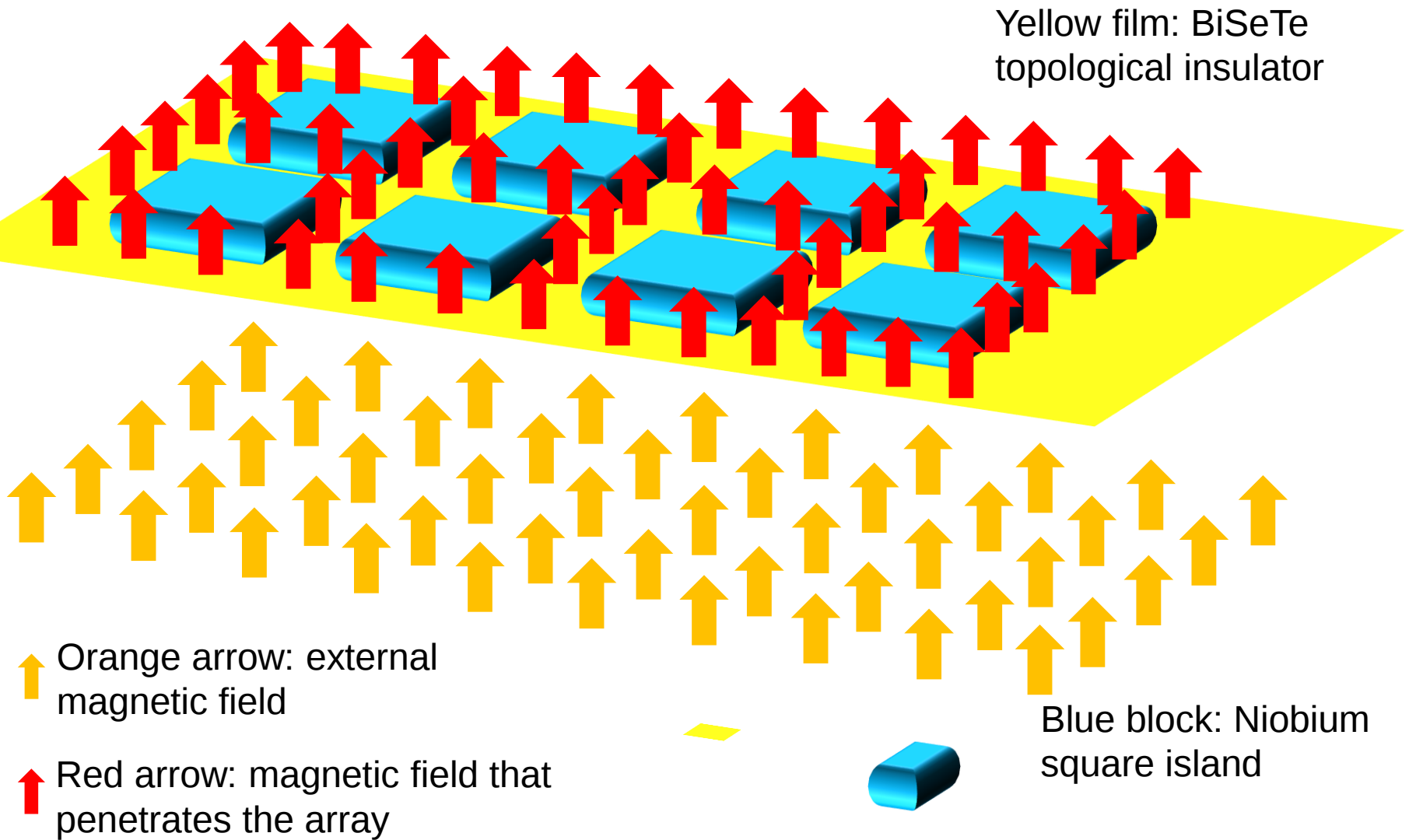
A. Bezryadin, Yu. Ovchinnikov, B. Pannetier, PRB 53, 8553 (1996)

R.S. Akzyanov, A.V. Rozhkov, A.L. Rakhmanov, and F. Nori, PRB 89, 085409 (2014)

PHYSICAL REVIEW B 84, 075141 (2011)



Superconducting array based on topological insulator- BST

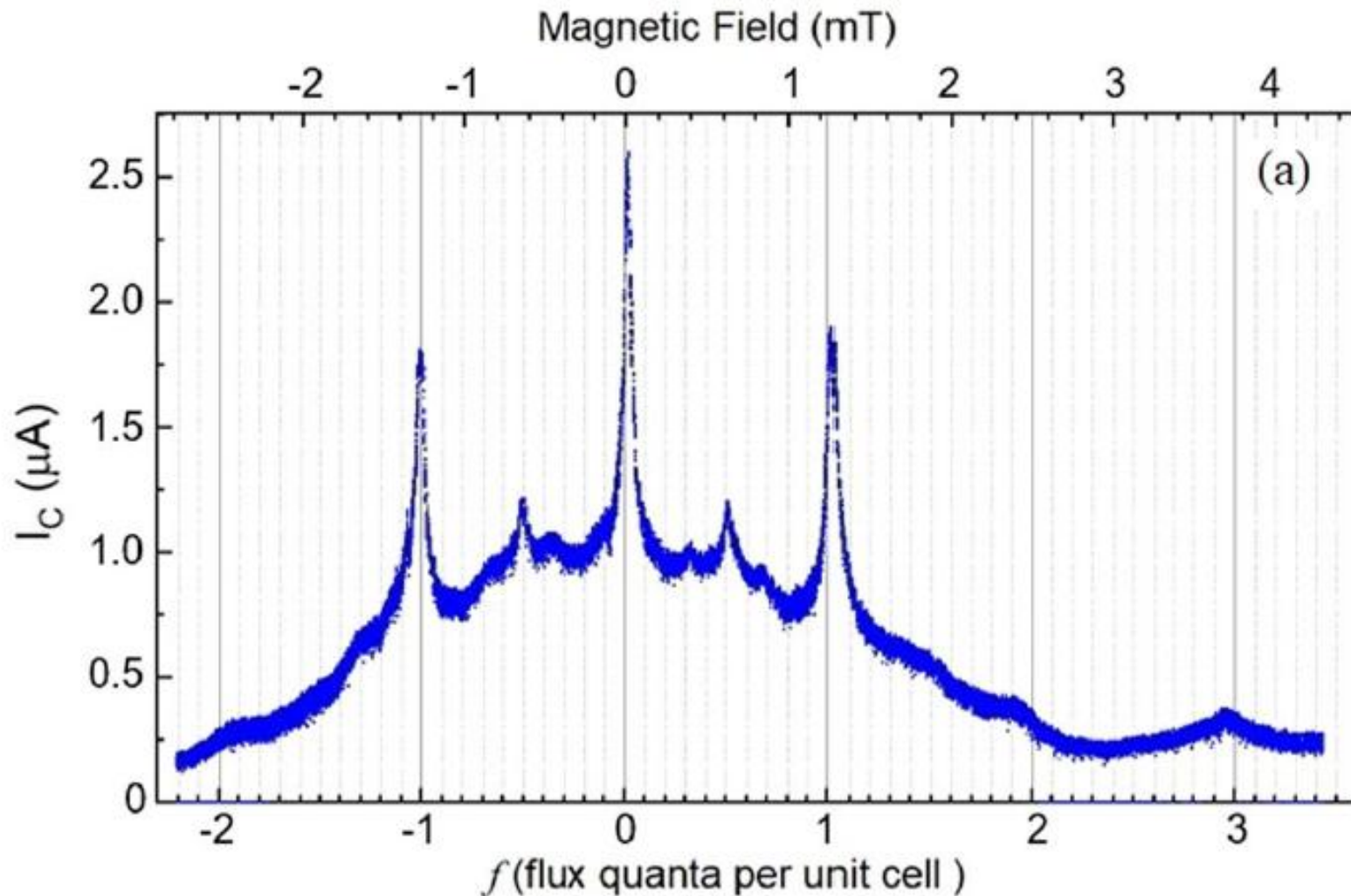


Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin

"Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator"

COMMUNICATIONS PHYSICS | (2023) 6:177 | <https://doi.org/10.1038/s42005-023-01288-9> | www.nature.com/commsphys

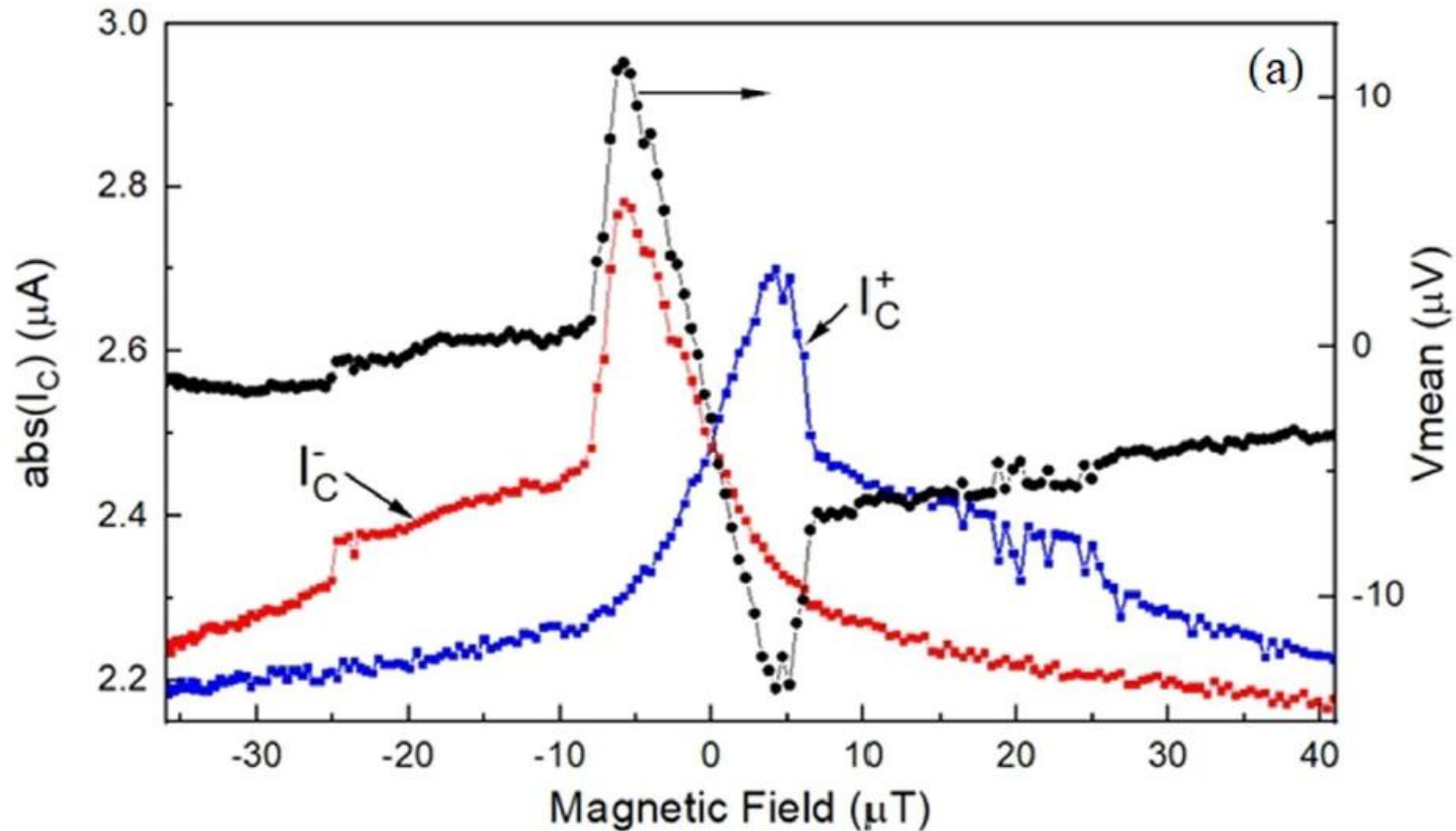
Diffraction grating analogy with superconducting arrays



Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin

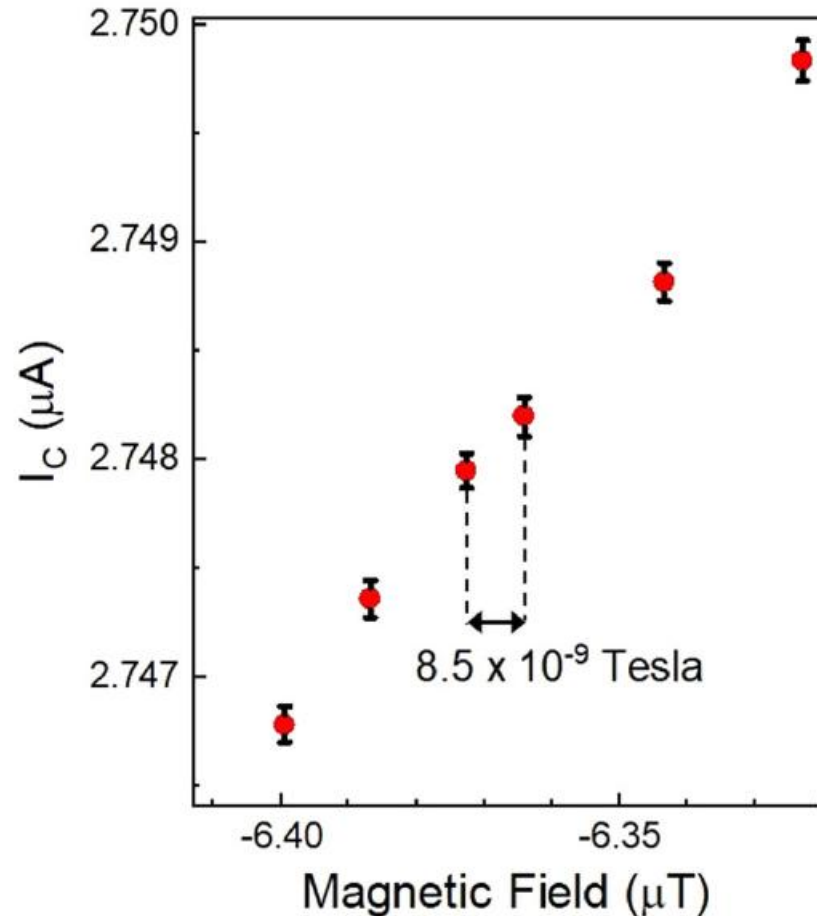
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
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Diode effect with superconducting arrays



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COMMUNICATIONS PHYSICS | (2023) 6:177 | <https://doi.org/10.1038/s42005-023-01288-9> | www.nature.com/commsphys

Absolute magnetic field sensor based on superconducting array



Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin

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